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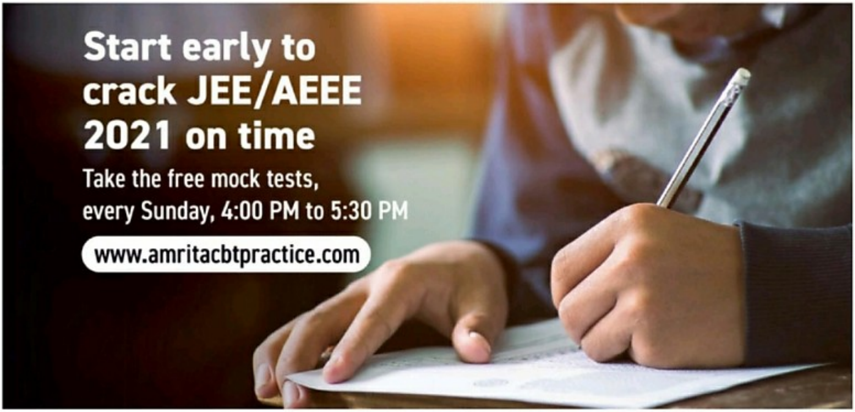
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# MOCK TEST PAPER 2021
# JEE MAIN

**Exam Dates 2021**
15th to 18th March
27th to 30th April
24th to 28th May

**Chapters covered : Sets, Relations and Functions, Trigonometry and Co-ordinate Geometry**

## SECTION-A (MULTIPLE CHOICE QUESTIONS)

**1.** If $(A \cap B) = \phi$, then $A \cup B'$ equals
(a) $A$ (b) $B$ (c) $A'$ (d) $B'$

**2.** Let $Y = \{1, 2, 3, 4, 5\}$, $A = \{1, 2\}$, $B = \{3, 4, 5\}$, then $(Y \times A) \cap (Y \times B)$ is
(a) $Y$ (b) $A$ (c) $B$ (d) $\phi$

**3.** Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$, then the relation is
(a) reflexive and transitive only
(b) reflexive only
(c) an equivalence relation
(d) reflexive and symmetric only

**4.** A relation $\rho$ on the set of real numbers $R$ is defined as follows: " $x\rho y$ if and only if $xy > 0$. Then which of the following is true?
(a) $\rho$ is reflexive and symmetric
(b) $\rho$ is symmetric and transitive
(c) $\rho$ is transitive only
(d) $\rho$ is an equivalence relation.

**5.** The range of the function

$$y = f(x) = \log_2 \frac{\sin x - \cos x + 3\sqrt{2}}{2} \text{ is}$$

(a) $0 < y \le 3$ (b) $0 < y < 3$
(c) $1 \le y \le 2$ (d) $1 < y < 2$

**6.** The domain of the function
$f(x) = \log_4 (\log_5(\log_3(18x - x^2 - 17)))$ is
(a) $x \in (4, 5)$ (b) $x \in (0, 10)$
(c) $x \in (8, 10)$ (d) None of these

**7.** The value of '$a$' for which $f : R \to R, f(x) = \dfrac{ax + 7}{x^2 + 4}$ is invertible is
(a) $(0, 2)$ (b) $(1, \infty)$
(c) $(0, \infty)$ (d) None of these

**8.** Let $f : R \to R$ where $f(x) = x^2 + 5x + 9$, then $f^{-1}(8) =$

(a) $\left\{\dfrac{-5+\sqrt{20}}{2}, \dfrac{-5-\sqrt{21}}{2}\right\}$

(b) $\left\{\dfrac{-5+\sqrt{21}}{2}, \dfrac{-5-\sqrt{21}}{2}\right\}$

(c) $\left\{\dfrac{5-\sqrt{21}}{2}, \dfrac{21-\sqrt{5}}{2}\right\}$

(d) None of these

**9.** Let $f(x) = \log|x|$ and $g(x) = \sin x$. If $A$ is the range of $f(g(x))$ and $B$ is the range of $g(f(x))$, then
(a) $A \cup B = (-\infty, 1]$ (b) $A \cup B = (-\infty, \infty)$
(c) $A \cap B = [-1, 1]$ (d) $A \cap B = [0, 1]$

**10.** The domain of $f(x) = \sqrt{\dfrac{4 - x^2}{2 + [x]}}$ is
(a) $[-2, 2]$ (b) $(-1, 2)$
(c) $(-\infty, -2) \cup [-1, 2]$
(d) $(-\infty, -2) \cup (-1, 2)$.

**11.** Let $f : (-1, 1) \to B$, $f(x) = \tan^{-1} \dfrac{2x}{1 - x^2}$ is one-one and onto, then $B =$

(a) $\left[0, \dfrac{\pi}{2}\right]$ (b) $\left(0, \dfrac{\pi}{2}\right)$

(c) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ (d) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

**12.** If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain

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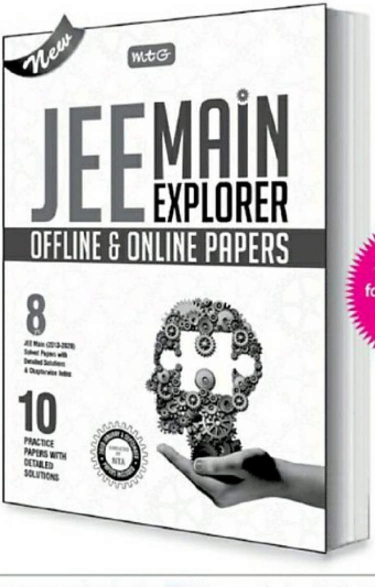
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(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $[0, \pi]$

**13.** The function $f(x)=\sqrt{3}\sin 2x-\cos 2x+4$ is one-one in the interval

(a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(c) $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$ (d) $[-\pi, \pi]$

**14.** The range of $f(x)=\frac{x-[x]}{1-[x]+x}$ is given by

(a) $[0,-1]$ (b) $\left[-\frac{1}{2}, 0\right]$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $\left[0, \frac{1}{2}\right]$

**15.** If $[\sin x]+[\sqrt{2}\cos x]=-3, x\in[0,2\pi]$ (where $[\cdot]$ denotes the greatest integer function), then $x$ belongs to

(a) $\left[\frac{5\pi}{4}, 2\pi\right]$ (b) $\left(\frac{5\pi}{4}, 2\pi\right)$

(c) $\left(\pi, \frac{5\pi}{4}\right)$ (d) $\left[\pi, \frac{5\pi}{4}\right]$

**16.** The smallest possible values of $x$ and $y$ which satisfy $\tan(x-y)=1$ and $\sec(x+y)=\frac{2}{\sqrt{3}}$ are

(a) $x=\frac{37\pi}{24}, y=\frac{7\pi}{24}$ (b) $x=\frac{15\pi}{24}, y=\frac{19\pi}{24}$

(c) $x=\frac{\pi}{3}, y=\frac{7\pi}{12}$ (d) $x=\frac{\pi}{4}, y=\frac{\pi}{2}$

**17.** In the interval $\left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$, the value of $\sqrt{\frac{1}{\cos^2\alpha}-\frac{2}{\cot\alpha}}=$

(a) $1-\tan\alpha$ (b) $\tan\alpha-1$

(c) $\cot\alpha-1$ (d) $1-\cot\alpha$

**18.** If the origin is shifted to the point $(-1, 2)$ without rotation, then the curve whose equation is $2x^2+y^2-4x+4y=0$ will have the equation

(a) $2x^2+y^2-8x+8y+18=0$

(b) $2x^2+y^2-6=0$

(c) $2x^2+y^2+8y+10=0$

(d) $x^2+2y^2+8x-8y+18=0$

**19.** If all the three vertices of an isosceles right angle triangle be integral points and length of base is also an integer, then which of the point is never a rational point. (A point $p(x, y)$ is integer point if both $x$ and $y$ are integers and is rational if both $x$ and $y$ are rational).

(a) centroid (b) incentre

(c) circumcentre (d) orthocentre

**20.** The co-ordinates of the third vertex of an equilateral triangle whose two vertices are $(3, 4)$, $(-2, 3)$ are

(a) $(1, 7)$ (b) $(5, 1)$

(c) $\left(\frac{1+\sqrt{3}}{2}, \frac{7-5\sqrt{3}}{2}\right)$ or $\left(\frac{1-\sqrt{3}}{2}, \frac{7+5\sqrt{3}}{2}\right)$

(d) None of these

## SECTION-B (NUMERICAL VALUE TYPE)

**Attempt any 5 questions out of 10.**

**21.** In a class of 100 students, 55 have passed in Mathematics and 67 in physics. If each of the students pass at least in one of the subjects, then the number of students who have passed in physics only is ______.

**22.** If $A$ is the set of even natural numbers less than 8 and $B$ is the set of prime numbers less than 7, then the number of relations from $A$ to $B$ is $2^{\lambda}$, where value of $\lambda$ is ______.

**23.** Let $f:[-3,3]\rightarrow R$, where $f(x)=x^3+\sin x+\left[\frac{x^2+2}{a}\right]$ be an odd function, then values of $a$ is greater than (where $[\cdot]$ represents greatest integer function) ______.

**24.** $f: R\rightarrow\left\{\frac{2}{3}\right\}\Rightarrow R$, $f(x)=\frac{3x+5}{2x-3}$. Let $f_2(x)=f(f(x)), f_3(x)=f(f_2(x)), \ldots, f_n(x)=f(f_{n-1}(x))$, then $f_{2018}(x)+f_{2019}(x)=\frac{ax^2+5}{ax-3}$, where value of $a$ is ______.

**25.** If $f:[1,\infty)\rightarrow[2,\infty)$, $f(x)=x+\frac{1}{x}$, then $f^{-1}(x)=ax+\frac{\sqrt{x^2-4}}{2}$, where value of $a$ is ______.

**26.** The period of the function $f(\theta)=\sin\frac{\theta}{3}+\cos\frac{\theta}{2}$ is $\lambda\pi$, then $\lambda$ is equal to ______.

**27.** If $e^{(\sin^2 x + \sin^4 x + \sin^6 x + \text{....to } \infty)\log_e 2}$ satisfies the equation $x^2 - 9x + 8 = 0$, then the value of $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$ is $\lambda(\sqrt{3}-1)$, where $\lambda$ is equal to ______.

**28.** If $\theta = \frac{\pi}{4n}$, where $n$ is positive integer, then the value of $\tan\theta \tan 2\theta \tan 3\theta \ldots \tan(2n - 1)\theta$ will be equal to ______.

**29.** $P, Q, R, S$ are the mid-points of the sides of a quadrilateral $ABCD$ in that order. If the co-ordinates of $P, Q, R, S$ are $(0, 1), (3, 5), (4, 6)$ and $(a, b)$ respectively, then $a + b$ equals ______.

**30.** If the vertices of a triangle are $\left(4, \frac{1}{4}\right), \left(3, \frac{1}{3}\right), (1,1)$, then orthocentre of the triangle is $\left(-\frac{1}{\lambda}, -\lambda\right)$, where $\lambda$ is equal to ______.

## SOLUTIONS

**1.** **(d) :** Here, $A \cup B' = (A \cup B') \cap U$

$= (A \cup B') \cap (A \cap B)'$

$[\because A \cap B = \phi \;\therefore\; (A \cap B)' = \phi' = U]$

$= (A \cup B') \cap (A' \cup B') = (A \cap A') \cup B'$

$= \phi \cup B' = B'$

**2.** **(d) :** $A = \{1, 2\}, B = \{3, 4, 5\}, A \cap B = \phi$

$(Y \times A) \cap (Y \times B) = Y \times (A \cap B) = Y \times \phi = \phi$

**3.** **(a) :** Given that, $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$

**Reflexive property :**

Since $(3, 3), (6, 6), (9, 9), (12, 12) \in R$ *i.e.*, $(a, a) \in R$ $\forall\, a \in A$ is satisfied. Hence, $R$ is reflexive.

**Symmetric property :**

Here, $(3, 6) \in R$ but $(6, 3) \notin R$

Hence, $R$ is not symmetric.

**Transitive property :**

Here, we find that $(3, 6)$ and $(6, 12) \in R \Rightarrow (3, 12) \in R$ *i.e.* $(a, b), (b, c) \in R \Rightarrow (a, c) \in R\; \forall\, a, b, c \in A$ is satisfied. Hence, $R$ is transitive.

**4.** **(b) : Reflexive property :**

$0 \in R$ (set of real numbers)

Then $(0, 0) \in \rho \Rightarrow 0 \cdot 0 > 0$ which is not true.

$\therefore\; \rho$ is not reflexive.

**Symmetric property :**

Let $x, y \in R$ such that $(x, y) \in \rho$.

$\Rightarrow xy > 0 \;\Rightarrow\; yx > 0 \;\Rightarrow\; (y, x) \in \rho$

Thus, $(x, y) \in \rho \;\Rightarrow\; (y, x) \in \rho$

Hence $\rho$ is symmetric.

**Transitive property :**

Let $x, y, z \in R$ such that $(x, y), (y, z) \in \rho$.

$\Rightarrow xy > 0$ and $yz > 0 \Rightarrow (xy)(yz) > 0 \;\Rightarrow\; xzy^2 > 0$

$\Rightarrow xz > 0 \;\Rightarrow\; (x, z) \in \rho$

Thus, $(x, y), (y, z) \in \rho \;\Rightarrow\; (x, z) \in \rho$

Hence, $\rho$ is transitive.

**5.** **(c) :** We have, $y = \log_2\left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$

$\Rightarrow\; \frac{1}{\sqrt{2}}(\sin x - \cos x) + 3 = 2^y$

$\Rightarrow\; \sin\left(x - \frac{\pi}{4}\right) + 3 = 2^y \;\Rightarrow\; \sin\left(x - \frac{\pi}{4}\right) = 2^y - 3$

We know, $-1 \le \sin\left(x - \frac{\pi}{4}\right) \le 1$

$\Rightarrow -1 \le 2^y - 3 \le 1 \;\Rightarrow\; 2 \le 2^y \le 4$

$\Rightarrow 2^1 \le 2^y \le 2^2 \;\Rightarrow\; 1 \le y \le 2$

**6.** **(c) :** We know $\log x$ is defined for all $x > 0$.

$\therefore\; f(x) = \log_4(\log_5(\log_3(18x - x^2 - 77)))$

$\Rightarrow \log_5(\log_3(18x - x^2 - 77)) > 0$

$\Rightarrow \log_3(18x - x^2 - 77) > 5^0$

$\Rightarrow 18x - x^2 - 77 > 3^1 \;\Rightarrow\; x^2 - 18x + 80 < 0$

$\Rightarrow (x - 8)(x - 10) < 0 \;\Rightarrow\; x \in (8, 10)$

**7.** **(d) :** Here, $f(x) = \frac{ax + 7}{x^2 + 4}$

$\therefore\; f'(x) = \frac{(x^2 + 4)a - (ax + 7)(2x)}{(x^2 + 4)^2} = \frac{-ax^2 - 14x + 4a}{(x^2 + 4)^2}$

Now, $f'(x) > 0$ for invertible function

$\therefore\; D = 196 - 4(4a)(-a) = 196 + 16a^2 > 0$

## VITEEE 2021 to be held in June

The Vellore Institute of Technology (VIT) Engineering Entrance Examination (VITEEE) which was scheduled to be held in April earlier has been postponed till June. Now, the engineering entrance exam will be conducted from June 18 to 26. The entrance test will be held after the last attempts of JEE Main schedule (in May).

Candidates who have not applied for VITEEE 2021 can do so at the official website – *viteee.vit.ac.in*. The application process had begun in November. Those who clear the exam will be eligible to seek admission in B.Tech programmes across VIT Vellore, Chennai, AP and Bhopal campuses.

VITEEE has multiple-choice questions (MCQs). It will have following sections- maths or biology (40 questions), physics (35 questions), chemistry (35 questions), aptitude (10 questions), and English (5 questions). Those attempting PCM will be eligible for Bachelor of Technology (B.Tech) courses. Those attempting PCB will be eligible for B.Tech Bio-Medical Engineering, B.Tech Biotechnology, and B.Tech Computer Science programmes.

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**8. (b) :** Given that $f : R \rightarrow R$ such that

$f(x) = x^2 + 5x + 9$

Now, $f^{-1}(8) = \{x : f(x) = 8\}$

Now, $f(x) = 8$

$\Rightarrow x^2 + 5x + 9 = 8 \Rightarrow x^2 + 5x + 1 = 0$

$$\Rightarrow x = \frac{-5+\sqrt{21}}{2}, \frac{-5-\sqrt{21}}{2} \in R$$

Hence, $f^{-1}(8) = \left\{\frac{-5+\sqrt{21}}{2}, \frac{-5-\sqrt{21}}{2}\right\}$

**9. (a) :** The range of $|\sin x| = [0, 1]$

Now, $A$ = range of $f(g(x))$

$\quad$ = range of $\log|\sin x| = (-\infty, 0]$

The range of $\log|x| = (-\infty, \infty)$

$B$ = range of $g(f(x))$

$\quad$ = range of $\sin(\log|x|) = [-1, 1]$

$\therefore \; A \cup B = (-\infty, 0] \cup [-1, 1] = (-\infty, 1]$

and $A \cap B = (-\infty, 0] \cap [-1, 1] = [-1, 0]$

**10. (c) :** Here, $f(x) = \sqrt{\frac{4-x^2}{2+[x]}}$

The function $f(x)$ will be defined if $\frac{4-x^2}{2+[x]} \geq 0$

**Case I :** $(4 - x^2) \geq 0$ and $2 + [x] > 0$

$\Rightarrow (x + 2)(x - 2) \leq 0$ and $[x] > -2$

$\Rightarrow x \in [-2, 2]$ and $x \in [-1, \infty)$

$\therefore \; x \in [-1, 2]$ ... (i)

**Case II :** $4 - x^2 < 0$ and $2 + [x] < 0$

$\Rightarrow x \in R - [-2, 2]$ and $x \in (-\infty, -2)$

$\Rightarrow x \in (-\infty, -2)$ ...(ii)

From (i) and (ii), $x \in (-\infty, -2) \cup [-1, 2]$, which is the required domain.

**11. (c) :** We have, $f(x) = \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x$

$\therefore \; B = f(-1, 1) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

**12. (b) :** Here $f(x) = \sin x + \cos x$ and $g(x) = x^2 - 1$

$\therefore \; g(f(x)) = (f(x))^2 - 1 = (\sin x + \cos x)^2 - 1 = \sin 2x$

We know that $\sin x$ is invertible in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \; \sin 2x$ is invertibe in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.

**13. (c) :** Here, $f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$

$$\Rightarrow f(x) = 2\left(\frac{\sqrt{3}}{2}\sin 2x - \frac{1}{2}\cos 2x\right) + 4$$

$$\Rightarrow f(x) = 2\sin\left(2x - \frac{\pi}{6}\right) + 4$$

We know that $\sin x$ is one-one in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore -\frac{\pi}{2} \leq 2x - \frac{\pi}{6} \leq \frac{\pi}{2} \Rightarrow -\frac{\pi}{3} \leq 2x \leq \frac{2\pi}{3}$$

$$\Rightarrow -\frac{\pi}{6} \leq x \leq \frac{\pi}{3} \Rightarrow x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$$

**14. (d) :** Here, $f(x) = \frac{x-[x]}{1-[x]+x} = \frac{x-[x]}{1+x-[x]}$

$\therefore \; f(x) = \frac{\{x\}}{1+\{x\}}$

Since, $0 \leq \{x\} < 1$

$\therefore \; 1 \leq 1 + \{x\} < 2$

Now, $1 + \{x\} = 1$ when $\{x\} = 0$

Thus, $f(x) \in \left[0, \frac{1}{2}\right]$

**15. (c) :** Given $[\sin x] + [\sqrt{2}\cos x] = -3$

$\therefore \; [\sin x] = -1$ and $[\sqrt{2}\cos x] = -2$

Now, $[\sin x] = -1$

$\Rightarrow -1 \leq \sin x < 0$

$\Rightarrow x \in (\pi, 2\pi)$ ... (i)

Again, $[\sqrt{2}\cos x] = -2 \Rightarrow -2 \leq \sqrt{2}\cos x < -1$

$$\Rightarrow -\sqrt{2} \leq \cos x < -\frac{1}{\sqrt{2}} \Rightarrow \cos x < -\frac{1}{\sqrt{2}}$$

$\Rightarrow x \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ ... (ii)

From (i) and (ii), we get $x \in \left(\pi, \frac{5\pi}{4}\right)$

**16. (a) :** Given $\tan(x - y) = 1$ and $\sec(x + y) = \frac{2}{\sqrt{3}}$

When, $\tan(x - y) = 1$, then $x - y = \frac{\pi}{4}, \frac{5\pi}{4}$

When, $\sec(x + y) = \frac{2}{\sqrt{3}}$, then $x + y = \frac{\pi}{6}, \frac{11\pi}{6}$

Now, $x + y > x - y$ ($x$, $y$ are positive)

We take $x + y = \frac{11\pi}{6}$ and $x - y = \frac{5\pi}{4}$

Solving, we get $x = \frac{37\pi}{24}$, $y = \frac{7\pi}{24}$

……

## Monthly Test Drive-10 CLASS XII ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (c) | **2.** | (a) | **3.** | (a) | **4.** | (a) | **5.** | (c) |
| **6.** | (c) | **7.** | (a,b,c,d) | **8.** | (a,c) | **9.** | (a,c) | **10.** | (a,b) |
| **11.** | (c) | **12.** | (b,d) | **13.** | (b,c) | **14.** | (a) | **15.** | (b) |
| **16.** | (a) | **17.** | (8) | **18.** | (4) | **19.** | (4) | **20.** | (2) |

**17. (b) :** We have $\sqrt{\frac{1}{\cos^2\alpha}-\frac{2}{\cot\alpha}}=\sqrt{\sec^2\alpha-2\tan\alpha}$

$=\sqrt{1+\tan^2\alpha-2\tan\alpha}=\sqrt{(1-\tan\alpha)^2}=|1-\tan\alpha|$

$=\tan\alpha-1\ \left(\because\ \tan\alpha>1 \text{ when } \alpha\in\left[\frac{5\pi}{4},\frac{3\pi}{2}\right]\right)$

**18. (a) :** Here, the transformation is

$x = x' - 1$ and $y = y' + 2$

The given equation becomes

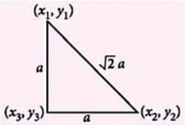
$2(x'-1)^2 + (y'+2)^2 - 4(x'-1) + 4(y'+2) = 0$

$\Rightarrow 2x'^2 + y'^2 - 8x' + 8y' + 18 = 0$

or $2x^2 + y^2 - 8x + 8y + 18 = 0$,

which is the required equation.

**19. (b) :** Since, $a$ is an integer, $\sqrt{2}a$ is irrational number.



Incentre will be $\left(\frac{ax_1+ax_2+\sqrt{2}ax_3}{2a+a\sqrt{2}},\frac{ay_1+ay_2+\sqrt{2}ay_3}{2a+a\sqrt{2}}\right)$,

which is clearly an irrational point.

**20. (c) :** Let $A(x, y)$, $B(3, 4)$ and $C(-2, 3)$.

Given that $\Delta ABC$ is an equilateral triangle.

$\therefore\ AB = AC = BC$

Now $AB = AC \Rightarrow AB^2 = AC^2$

$\Rightarrow (x-3)^2 + (y-4)^2 = (x+2)^2 + (y-3)^2$

$\Rightarrow 5x + y = 6$ or $y = 6 - 5x$ ...(i)

Again, $AC = BC$ gives

$(x+2)^2 + (y-3)^2 = 26$

$\Rightarrow (x+2)^2 + (3-5x)^2 = 26$ [Using (i)]

$\Rightarrow 2x^2 - 2x - 1 = 0 \Rightarrow x=\frac{1\pm\sqrt{3}}{2}$

Now from (i), we get $y=\frac{7\mp5\sqrt{3}}{2}$

**21. (45) :** Let $A$ be the set of students who passed in Mathematics and $B$ be the set of students who passed in Physics.

$\therefore\ n(A\cup B) = 100$

Now, $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$\because\ n(A) = 55$ and $n(B) = 67$

$\Rightarrow 100 = 55 + 67 - n(A\cap B)$

$\Rightarrow n(A\cap B) = 122 - 100 = 22$

$\therefore$ The number of students who have passed in Physics only $= n(B) - n(A\cap B) = 67 - 22 = 45$

**22. (9) :** Here, $n(A) = 3$ and $n(B) = 3$

$\therefore$ Number of relations $= 2^{3\times3} = 2^9$

**23. (11) :** Given: $f(x) = x^3 + \sin x + \left[\frac{x^2+2}{a}\right]$ ... (i)

$\therefore\ f(-x)=-x^3-\sin x+\left[\frac{x^2+2}{a}\right]$ ... (ii)

Now, $f(x) = -f(-x)$ [$\because f(x)$ is an odd function]

$\Rightarrow f(x) + f(-x) = 0 \Rightarrow 2\left[\frac{x^2+2}{a}\right]=0$

$\Rightarrow \left[\frac{x^2+2}{a}\right]=0 \Rightarrow 0\le\frac{x^2+2}{a}<1\ \forall\, x\in[-3, 3]$

$\therefore\ a > 11$ [$\because$ Maximum of $x^2 + 2$ in $[-3, 3]$ is 11]

**24. (2) :** Here $f(x)=\frac{3x+5}{2x-3}$

Now, $f_2(x)=f(f(x))=\frac{3f(x)+5}{2f(x)-3}=x$

$\Rightarrow f_2(x) = f_4(x) = .... = f_{2018}(x) = x$

and $f(x) = f_3(x) = .... = f_{2019}(x) = \frac{3x+5}{2x-3}$

$\therefore\ f_{2018}(x)+f_{2019}(x)=x+\frac{3x+5}{2x-3}=\frac{2x^2+5}{2x-3}$

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**Form IV**

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I, Mahabir Singh, authorised signatory for MTG Learning Media Pvt. Ltd. hereby declare that particulars given above are true to the best of my knowledge and belief.

For MTG Learning Media Pvt. Ltd.
Mahabir Singh
Director

**25. (0.5) :** Given that $f: [1, \infty) \to [2, \infty),\ f(x) = x + \frac{1}{x}$

Let $y \in [2, \infty)$

$\therefore\ f(x) = y \Rightarrow x + \frac{1}{x} = y$

$\Rightarrow\ x^2 - yx + 1 = 0 \quad \Rightarrow \quad x = \frac{y \pm \sqrt{y^2 - 4}}{2}$

Clearly $\forall\ y \in [2, \infty)$, there exist an unique $x$ in $[1, \infty)$. Thus $f$ is onto.

Clearly, for $x_1, x_2 \in [1, \infty), f(x_1) = f(x_2)$

$$\Rightarrow \quad x_1 + \frac{1}{x_1} = x_2 + \frac{1}{x_2} \quad \Rightarrow \quad (x_1 - x_2)\left(1 - \frac{1}{x_1 x_2}\right) = 0$$

$$\Rightarrow \quad x_1 = x_2 \quad \left[\because 1 - \frac{1}{x_1 x_2} \neq 0\right]$$

So, $f$ is one-one function.

Thus, $f$ is one-one and onto, hence $f$ is bijective.

Since, $f$ is bijective, so it is invertible.

$$\therefore \quad f^{-1}(y) = x = \frac{y + \sqrt{y^2 - 4}}{2} = \frac{x}{2} + \frac{\sqrt{x^2 - 4}}{2}$$

$$\text{or} \quad f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2} = \frac{x}{2} + \frac{\sqrt{x^2 - 4}}{2}$$

$$\left[\because \ f^{-1}(\sqrt{5}) = \frac{\sqrt{5} - 1}{2} < 1 \text{ if negative sign is used}\right]$$

**26. (12) :** Period of $\sin\frac{\theta}{3} = \frac{2\pi}{1/3} = 6\pi$

and period of $\cos\frac{\theta}{2} = \frac{2\pi}{1/2} = 4\pi$

$\therefore$ Period of $f(\theta)$ = L.C.M. $(6\pi, 4\pi) = 12\pi$

**27. (0.5) :** $e^{(\sin^2 x + \sin^4 x + \sin^6 x + \text{.... to } \infty)\log_e 2}$

$$= e^{\left(\frac{\sin^2 x}{1 - \sin^2 x}\right)\log_e 2} = e^{\tan^2 x \log_e 2} = e^{\log_e 2^{\tan^2 x}} = 2^{\tan^2 x}$$

Now, $x^2 - 9x + 8 = 0 \quad \Rightarrow \quad x = 1$ and $x = 8$

$\therefore\ 2^{\tan^2 x} = 1$ or $8 \Rightarrow 2^{\tan^2 x} = 2^0$ or $2^3$

If $\tan^2 x = 0 \ \Rightarrow\ x = 0$ or $\tan^2 x = 3 \Rightarrow\ x = \pm\frac{\pi}{3}$

But only $\frac{\pi}{3}$ belongs to $0 < x < \frac{\pi}{2}$ $\ \therefore\ x = \frac{\pi}{3}$

**28. (1) :** Given $\theta = \frac{\pi}{4n}\ \therefore\ 2n\theta = \frac{\pi}{2}$

Now, $\tan(2n - 1)\theta = \tan(2n\theta - \theta) = \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

Also, $\tan(2n - 2)\theta = \tan(2n\theta - 2\theta)$

$$= \tan\left(\frac{\pi}{2} - 2\theta\right) = \cot 2\theta$$

and so on.

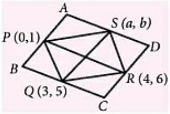
Now, $\tan\theta \tan2\theta \tan3\theta\ ....\ \tan(2n - 1)\theta$

$= \tan\theta \tan2\theta \tan3\theta\ ....\ \tan(2n - 3)\theta \tan(2n - 2)\theta \times \tan(2n - 1)\theta$

$= \tan\theta \tan2\theta \tan3\theta\ ...\ \cot3\theta \cot2\theta \cot\theta$

$= (\tan\theta \cot\theta)(\tan2\theta \cot2\theta)(\tan3\theta \cot3\theta)\ ....$

$= 1 \cdot 1 \cdot 1\ .... = 1$



**29. (3) :** Clearly, $P, Q, R$ and $S$ are the vertices of parallelogram $PQRS$. We know diagonals of a parallelogram bisect each other.

Hence, $\left(\frac{0+4}{2}, \frac{1+6}{2}\right) \equiv \left(\frac{3+a}{2}, \frac{5+b}{2}\right)$

$\Rightarrow\ \frac{3+a}{2} = 2 \Rightarrow a = 1$ and $\frac{5+b}{2} = \frac{7}{2} \Rightarrow b = 2$

**30. (12) :** We know that, if vertices of a $\Delta ABC$ are $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$, then the co-ordinate of orthocentre is $\left(-\frac{1}{abc}, -abc\right)$.

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# PUBLIC NOTICE

## Date Clash of JEE (Main) May 2021 with Class 12 Board Examination

The JEE (Main) - 2021 is being conducted in **multiple sessions** (February/March/April/May) for admissions in the next academic session. The JEE (Main) **Session-1 (February-2021), Session-2 (March-2021), and Session-3 (April-2021) are being held from 23rd to 26th February 2021, 15th to 18th March 2021 and 27th to 30th April 2021 respectively.**

The JEE (Main) **Session-4** will be conducted in May on **24th, 25th, 26th, 27th and 28th May 2021.** It has been brought to the notice of NTA by the candidates appearing in JEE (Main) that the 12th Class Examinations, 2021 conducted by CBSE and various State Boards are going to be held during this period.

Keeping in view the interest of the candidates and to avoid any clash between the two examinations, the NTA will open the Application form of JEE(Main) - 2021 from **3rd May to 12th May 2021 (for the May session)** asking the candidates to inform the NTA about their Class 12 Roll number and name of Board.

The candidates appearing for the Class 12 Board examination as well as JEE (Main) will be required to provide information under "Whether appearing for Board Examination on any of the dates **24th, 25th, 26th, 27th and 28th May 2021**" and select the 'Date of their Class 12 Board Examination' in their online Application Form so that there is no clash of their schedule of JEE (Main) with Class 12 Board Examination.

For more updates or the latest news, keep visiting the website of NTA **www.nta.ac.in** and **jeemain.nta.nic.in.**

# How to improve your marks in Maths for IIT JEE?

**Vikas Gupta**

Co-founder, Author, Director
and HOD of Mathematics at
Vibrant Academy

Mr Vikas Gupta is the Co-founder, Director and HOD of Mathematics at Vibrant Academy. He has 18+ years of experience in teaching Mathematics and has authored several famous books like Advanced Problems in Mathematics, Advanced Problems in Coordinate Geometry & Advanced Problems in Algebra. Here's what he has to say about improving your marks in Mathematics for IIT JEE.

When you start solving the paper, this is the approach that you should follow—see if the question can be solved easily. In case the method to solve it doesn't strike you immediately, then highlight the question and proceed to the next.

There are certain problems which at first may seem easy to solve, but somewhere in the middle of the calculations, you may get stuck and it may take more time than usual. In this case, highlight the problem separately, leave it for the second attempt and move forward to the next question.

After solving a question, make it a habit to go through it once again. This will hardly take a few seconds. This exercise should be done after the completion of every problem and will help you improve your accuracy and find silly mistakes (if any). It is also very helpful in tracing conceptual errors in the solution.

The next step is to continue with the same approach till the last problem of the paper and finish the first attempt before the time limit. This way you won't get stuck with a problem and it will help you save a lot of time.

After the first attempt, you should go back for a second attempt and take care of the highlighted problems which you think you will be able to solve. The last attempt should be for the problems that you couldn't figure out earlier. Often in the second attempt, you will be able to solve the question very easily. The reason behind this is that after solving many questions, you gain confidence and the ideas you have applied in solving other problems gives you perspective to solve the tricky questions.

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Exam Dates 2021
15th to 18th March
27th to 30th April
24th to 28th May

# FOCUS JEE MAIN 2021

# PRACTICE QUESTIONS

**1.** If $2^{\sqrt{\sin^2 x-2\sin x+5}}\,\frac{1}{4^{\sin^2 y}}\le 1$, then the ordered pair $(x, y)$ is equal to $(m, n \in I)$

(a) $x=(4n+1)\frac{\pi}{2}, y=(2m+1)\frac{\pi}{2}$

(b) $x = 2n\pi$, $y = 2m\pi$

(c) $x=(2n+1)\frac{\pi}{2}, y=(2m+1)\frac{\pi}{2}$

(d) $x = n\pi$, $y = m\pi$

**2.** If $a\sin^2 x + b\cos^2 x = c$, $b\sin^2 y + a\cos^2 y = d$ and $a\tan x = b\tan y$, then $\frac{a^2}{b^2}=$

(a) $\frac{(a-d)(c-a)}{(b-c)(d-b)}$ (b) $\frac{(a+d)(c+a)}{(b+c)(d+b)}$

(c) $\frac{(a-d)(b-a)}{(a-c)(c-b)}$ (d) $\frac{(d-a)(c-a)}{(b-c)(d-b)}$

**3.** If $\cos\theta=\frac{a\cos\phi+b}{a+b\cos\phi}$, then $\tan\frac{\theta}{2}$ is equal to

(a) $\sqrt{\left(\frac{a-b}{a+b}\right)}\tan(\phi/2)$ (b) $\sqrt{\left(\frac{a+b}{a-b}\right)}\cos(\phi/2)$

(c) $\sqrt{\left(\frac{a-b}{a+b}\right)}\sin(\phi/2)$ (d) none of these

**4.** A set of values of $x$, satisfying the equation $\cos^2\left(\frac{1}{2}px\right)+\cos^2\left(\frac{1}{2}qx\right)=1$ form an arithmetic progression with common difference

(a) $\frac{2}{p+q}$ (b) $\frac{2}{p-q}$

(c) $\frac{\pi}{p+q}$ (d) none of these

**5.** If $\cos^6\alpha+\sin^6\alpha+k\sin^2 2\alpha=1\ \forall\ \alpha\in(0,\pi/2)$, then $k$ is

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$

**6.** The most general solution of the equations $\tan\theta=-1$, $\cos\theta=\frac{1}{\sqrt{2}}$ is

(a) $n\pi + 7\pi/4$ (b) $n\pi+(-1)^n\frac{7\pi}{4}$

(c) $2n\pi+\frac{7\pi}{4}$ (d) none of these

**7.** The least positive values of $x$ satisfying the equation $8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+\ldots\infty}=4^3$ will be (where $|\cos x|<1$)

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{4}$ (d) none of these

**8.** For each real number $x$ such that $-1 < x < 1$, let $A(x)$ be the matrix $(1-x)^{-1}\begin{bmatrix}1 & -x\\ -x & 1\end{bmatrix}$ and $z=\frac{x+y}{1+xy}$. Then,

(a) $A(z) = A(x) + A(y)$ (b) $A(z) = A(x)\,[A(y)]^{-1}$

(c) $A(z) = A(x)\,A(y)$ (d) $A(z) = A(x) - A(y)$

**9.** If $A$ and $B$ are square matrices of the same order and $A$ is non-singular, then for a positive integer $n$, $(A^{-1}BA)^n$ is equal to

(a) $A^{-n}B^nA^n$ (b) $A^nB^nA^{-n}$
(c) $A^{-1}B^nA$ (d) $n(A^{-1}BA)$

**10.** If $A=\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then $A^8$ equals

(a) $4B$ (b) $128B$ (c) $-128B$ (d) $-64B$

**11.** If $p+q+r=0$ and $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}=k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, then $k=$

(a) 0 (b) $abc$ (c) $pqr$ (d) $a+b+c$

**12.** Let $A$, $B$ be square matrix such that $AB=O$ and $B$ is non singular, then

(a) $|A|$ must be zero but $A$ may non zero
(b) $A$ must be zero matrix
(c) nothing can be said in general about $A$
(d) none of these

**13.** $A=[a_{ij}]_{m\times n}$ and $a_{ij}=i^2-j^2$, then $A$ is necessarily

(a) a unit matrix (b) symmetric matrix
(c) skew symmetric matrix
(d) zero matrix

**14.** If $A=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, then $A^{16}=$

(a) $\begin{bmatrix} 0 & 256 \\ 256 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$
(c) $\begin{bmatrix} -16 & 0 \\ 0 & -16 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 16 \\ 16 & 0 \end{bmatrix}$

**15.** If $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $a, b\in N$, then number of matrix '$B$' such that $AB=BA$ are

(a) 0 (b) 1
(c) finitely many (d) infinite

**16.** Let $A$ and $B$ are two non-singular square matrices, and $A^T$ and $B^T$ are the transpose matrices of $A$ and $B$ respectively, then which of the following is correct?

(a) $B^TAB$ is symmetric matrix if and only if $A$ is symmetric
(b) $B^TAB$ is symmetric matrix if and only if $B$ is symmetric
(c) $B^TAB$ is skew symmetric matrix for every matrix $A$
(d) $B^TAB$ is skew symmetric matrix if $B$ is skew symmetric

**17.** $|A_{3\times 3}|=3$, $|B_{3\times 3}|=-1$, and $|C_{2\times 2}|=2$, then $|2ABC|=$

(a) $2^3(6)$ (b) $2^3(-6)$
(c) $2(-6)$ (d) none of these

**18.** Let $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are two matrices such that $AB=BA$ and $c\neq 0$, then value of $\frac{a-d}{3b-c}$ is

(a) 0 (b) 2 (c) $-2$ (d) $-1$

**19.** Let $f(\alpha)=\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $(f(\alpha))^{-1}$ is equal to

(a) $f(\alpha)$ (b) $f(-\alpha)$
(c) $f(\alpha-1)$ (d) none of these

**20.** $f: R\to R$, $f(x)=x|x|$ is

(a) one-one but not onto
(b) onto but not one-one
(c) Both one-one and onto
(d) neither one-one nor onto

**21.** The domain of the function

$f(x)=\sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right)$ is

(a) $[-2, 2]$ (b) $[-2, -1]$
(c) $[1, 2]$ (d) $[-2, -1]\cup[1, 2]$

**22.** The range of $f(x)=\frac{3}{5+4\sin 3x}$ is

(a) $\left[\frac{1}{3}, 3\right]$ (b) $\left[\frac{1}{3}, 1\right]$
(c) $[1, 3]$ (d) $\left(-\infty, \frac{1}{3}\right)\cup(3, \infty)$

**23.** The range of $\frac{x^2-x+1}{x^2+x+1}$ is

(a) $\left[\frac{1}{3}, 3\right]$ (b) $\left[\frac{1}{3}, 1\right]$
(c) $[1, 3]$ (d) $(-\infty, \frac{1}{3}]\cup[3, \infty)$

**24.** Domain of the function $f(x)=\sqrt{5|x|-x^2-6}$ is

(a) $(-\infty, 2)\cup(3, \infty)$ (b) $[-3, -2]\cup[2, 3]$
(c) $(-\infty, -2)\cup(2, 3)$ (d) $R-\{-3, -2, 2, 3\}$

**25.** Range of the function $f(x)=x^2+\dfrac{1}{x^2+1}$, is

(a) $[1,\infty)$ (b) $[2,\infty)$

(c) $\left[\dfrac{3}{2},\infty\right)$ (d) $(-\infty,\infty)$

**26.** The inverse of $f(x) = (5-(x-8)^5)^{1/3}$ is

(a) $5-(x-8)^5$ (b) $8+(5-x^3)^{1/5}$

(c) $8-(5-x^3)^{1/5}$ (d) $(5-(x-8)^{1/5})^3$

**27.** The domain of the function $f(x) = \log_{10}\{1-\log_{10}(x^2-5x+10)\}$ is

(a) $(0,\infty)$ (b) $(0,5)$

(c) $(-\infty,0)$ (d) None of these

**28.** The domain of definition of the function, $f(x)$ given by the equation $2^x+2^y=2$ is

(a) $0<x\le 1$ (b) $0\le x\le 1$

(c) $-\infty<x\le 0$ (d) $-\infty<x<1$

**29.** Let $f: R\to R-\{3\}$ be a function such that for some $p>0$, $f(x+p)=\dfrac{f(x)-5}{f(x)-3}$ for all $x\in R$. Then, period of $f$ is

(a) $2p$ (b) $3p$ (c) $4p$ (d) $5p$

**30.** If $f(x)=x-\dfrac{1}{x}$, then number of solutions of $f(f(f(x)))=1$ is

(a) 1 (b) 4 (c) 6 (d) 2

**31.** Let $f:\left[\dfrac{-\pi}{3},\dfrac{2\pi}{3}\right]\to[0,4]$ be a function defined as $f(x)=\sqrt{3}\sin x-\cos x+2$, then $f^{-1}(x)$ is given by

(a) $\sin^{-1}\left(\dfrac{x-2}{2}\right)-\dfrac{\pi}{6}$ (b) $\sin^{-1}\left(\dfrac{x+2}{2}\right)+\dfrac{\pi}{6}$

(c) $\dfrac{2\pi}{3}-\cos^{-1}\left(\dfrac{x-2}{2}\right)$ (d) Does not exist

**32.** The value of the parameter $\alpha$, for which the function $f(x) = 1+\alpha x$, $\alpha\neq 0$, is the inverse of itself, is

(a) $-2$ (b) $-1$ (c) 1 (d) 2

**33.** If for non zero $x$, $2f(x^2)+3f\left(\dfrac{1}{x^2}\right)=x^2-1$, then $f(x^2)=$

(a) $\dfrac{3+2x^4-x^2}{5x^2}$ (b) $\dfrac{3-2x^4+x^2}{5x^2}$

(c) $\dfrac{3-2x^4-x^2}{5x^2}$ (d) $\dfrac{3+2x^4+x^2}{5x^2}$

**34.** A function $f: R\to R$ satisfies the equation $f(x)f(y)-f(xy)=x+y\ \forall\ x, y\in R$ and $f(1)>0$, then

(a) $f(x)f^{-1}(x)=x^2-4$ (b) $f(x)f^{-1}(x)=x^2-6$

(c) $f(x)f^{-1}(x)=x^2-1$ (d) none of these

**35.** If $A$ and $B$ are two matrices such that $AB=B$ and $BA=A$, then

(a) $(A^6-B^5)^3=A-B$ (b) $(A^5-B^5)^3=A^3-B^3$

(c) $A-B$ is idempotent (d) $A-B$ is nilpotent

## Numerical Value Type

**36.** If $2\tan^2x-5\sec x=1$ for exactly 7 distinct values of $x\in\left[0,\dfrac{n\pi}{2}\right], n\in N$, then the greatest value of $n$ is ____.

**37.** Let $\theta\in[0,4\pi]$ satisfying the equation $(\sin\theta+2)(\sin\theta+3)(\sin\theta+4)=6$. If the sum of all values of $\theta$ is $K\pi$ then value of $K$ is ______.

**38.** The number of solutions of the equation $16(\sin^5x+\cos^5x)=11(\sin x+\cos x)$ in the interval $[0,2\pi]$ is ______.

**39.** Number of solutions of the equation $\tan x+\sec x=2\cos x$ lying in the interval $[0,2\pi]$ is ____.

**40.** If $\log_{0.5}\sin x=1-\log_{0.5}\cos x$, then number of values of $x\in[-2\pi,2\pi]$ is ______.

**41.** The value of $\cos^2 10^\circ-\cos 10^\circ\cos 50^\circ+\cos^2 50^\circ=$ ______.

**42.** Number of ordered pairs $(a, x)$ satisfying the equation $\sec^2(a+2)x+a^2-1=0;\ -\pi<x<\pi$ is _____.

**43.** If in a triangle, $\dfrac{s-a}{11}=\dfrac{s-b}{12}=\dfrac{s-c}{13}$ and $\lambda\tan^2(A/2)=455$, then $\lambda$ must be ______.

**44.** If $2\sin x-\cos 2x=1$, then $\cos^2x+\cos^4x$ is equal to ______.

**45.** If $A$ is a square matrix of order 3 such that $|A|=2$ then $|(\text{adj }A^{-1})^{-1}|$ is ______.

**46.** A square matrix $P$ satisfies $P^2=I-P$, where $I$ is an identity matrix of order as order of $P$. If $P^n=5I-8P$, then $n=$ ______.

**47.** The digits $A$, $B$, $C$ are such that the three digit numbers $A88$, $6B8$, $86C$ are divisible by 72, then the determinant $\begin{vmatrix}A & 6 & 8\\ 8 & B & 6\\ 8 & 8 & C\end{vmatrix}$ is divisible by ______.

**48.** If $A=\begin{bmatrix}5 & -6\\ 1 & -1\end{bmatrix}$ then determinant of $A^{1003}-5A^{1002}$ is ______.

**49.** The period of the function $f(x) = (-1)^{[x]}$, where $[\cdot]$ = G.I.F is ______.

**50.** If $g(x)$ is a polynomial satisfying $g(x)\, g(y) = g(x) + g(y) + g(xy) - 2$ for all real $x$ and $y$ and $g(2) = 5$, then $g(3)$ is equal to ______.

## SOLUTIONS

**1. (c) :** $\because \sin^2x - 2\sin x + 5 = (\sin x - 1)^2 + 4 \geq 4$

$\therefore\ 2^{\sqrt{\sin^2 x-2\sin x+5}} \geq 2^2 = 4$ and $\sin^2 y \leq 1 \Rightarrow \dfrac{1}{4^{\sin^2 y}} \geq \dfrac{1}{4}$

$\therefore$ L.H.S. $\geq 1$ and according to question L.H.S. $\leq 1$ therefore, L.H.S. = 1

for which $\sin^2x - 2\sin x + 5 = 4 \Rightarrow (\sin x - 1)^2 = 0$

$\Rightarrow\ \sin x = 1\ \Rightarrow\ x = (2n + 1)\dfrac{\pi}{2}$

and $\sin^2y = 1$ or $\cos y = 0 \Rightarrow y = (2m + 1)\dfrac{\pi}{2}$

**2. (a) :** $a\tan^2x + b = c(1 + \tan^2x)$

$\Rightarrow \tan^2 x=\left(\dfrac{c-b}{a-c}\right), \tan^2 y=\left(\dfrac{d-a}{b-d}\right)$

$\therefore\ \dfrac{a^2}{b^2}=\dfrac{\tan^2 y}{\tan^2 x}=\dfrac{(a-d)(c-a)}{(b-c)(d-b)}$

**3. (a) :** $\tan\theta/2=\sqrt{\left(\dfrac{1-\cos\theta}{1+\cos\theta}\right)}=\sqrt{\dfrac{1-\left(\dfrac{a\cos\phi+b}{a+b\cos\phi}\right)}{1+\left(\dfrac{a\cos\phi+b}{a+b\cos\phi}\right)}}$

$=\sqrt{\dfrac{(a-b)(1-\cos\phi)}{(a+b)(1+\cos\phi)}}=\sqrt{\dfrac{(a-b)}{(a+b)}}\tan(\phi/2)$

**4. (d) :** $1 + \cos px + 1 + \cos qx = 2$

$\Rightarrow \cos\left(\dfrac{p+q}{2}\right)x\cos\left(\dfrac{p-q}{2}\right)x=0$

$\Rightarrow x=\dfrac{(2n+1)\pi}{p+q}$ or $\dfrac{(2n+1)\pi}{p-q}$

for $n = 0, \pm1, \pm2, \ldots$..

forms an A.P. with common difference $\dfrac{2\pi}{p+q}$ or $\dfrac{2\pi}{p-q}$

**5. (a) :** The given condition can be written as

$(\cos^2\alpha + \sin^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha(\cos^2\alpha + \sin^2\alpha) + k\sin^22\alpha = 1$

$\Rightarrow\ \left(-\dfrac{3}{4}\right)\sin^2 2\alpha+k\sin^2 2\alpha=0$

Showing that $k = 3/4$.

**6. (c) :** We have, $\tan\theta = -1$ and $\cos\theta=\dfrac{1}{\sqrt{2}}$

The value of $\theta$ lying between $\dfrac{3\pi}{2}$ and $2\pi$ and satisfying these two is $\dfrac{7\pi}{4}$. Therefore the most general solution is $\theta = 2n\pi + 7\pi/4$ where $n \in Z$

**7. (a) :** $\because\ 1 + |\cos x| + \cos^2x\ldots = \dfrac{1}{1-|\cos x|}$

$\Rightarrow\ \dfrac{1}{8^{1-|\cos x|}}=4^3 \Rightarrow\ 2^{\frac{3}{1-|\cos x|}}=2^6$

$\Rightarrow\ \dfrac{3}{1-|\cos x|}=6 \Rightarrow 1-|\cos x|=\dfrac{1}{2}$

$\Rightarrow\ |\cos x|=\dfrac{1}{2}\ \Rightarrow\ \cos x=\pm\dfrac{1}{2}$

For least positive value of $x$, $x=\dfrac{\pi}{3}$

**8. (c) :** $A(z)=A\left(\dfrac{x+y}{1+xy}\right)$

$$=\left[\frac{1+xy}{(1-x)(1-y)}\right]\begin{bmatrix}1 & -\left(\dfrac{x+y}{1+xy}\right)\\ -\left(\dfrac{x+y}{1+xy}\right) & 1\end{bmatrix}$$

$\therefore\ A(x)\cdot A(y) = A(z)$

**9. (c) :** $(A^{-1}BA)^2 = (A^{-1}BA)(A^{-1}BA) = A^{-1}B(AA^{-1})BA$
$= A^{-1}BIBA = A^{-1}B^2A$

$\Rightarrow\ (A^{-1}BA)^3 = (A^{-1}B^2A)\,(A^{-1}BA) = A^{-1}B^2(AA^{-1})BA$
$= A^{-1}B^2IBA = A^{-1}B^3A$ and so on

$\Rightarrow\ (A^{-1}BA)^n = A^{-1}B^nA$

**10. (b) :** We have, $A = iB$

$\Rightarrow\ A^2 = (iB)^2 = i^2B^2 = -B^2 = -\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}=-2B$

$\Rightarrow\ A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$\Rightarrow\ (A^4)^2 = (8B)^2\ \Rightarrow\ A^8 = 64B^2 = 128B$

**11. (c) :** $p + q + r = 0$

$\Rightarrow\ p^3 + q^3 + r^3 = 3pqr$

Now, $\begin{vmatrix}pa & qb & rc\\ qc & ra & pb\\ rb & pc & qa\end{vmatrix}=pqr(a^3+b^3+c^3-3abc)$

$=pqr\begin{vmatrix}a & b & c\\ c & a & b\\ b & c & a\end{vmatrix}\Rightarrow k=pqr$

**12. (b) :** $A\cdot B = 0 \Rightarrow A\cdot B\cdot B^{-1} = 0\cdot B^{-1}$

$\Rightarrow\ A\cdot I = 0 \Rightarrow\ A = 0$

**13. (c)** : $a_{ji} = j^2 - i^2 = -(i^2 - j^2) = -a_{ij}$

**14. (b)** : $A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}, A^4 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$

$A^8 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}, A^{16} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

**15. (d)** : $AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}, BA = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

Given, $AB = BA \Rightarrow a = b$

**16. (a)** : $(B^TAB)^T = B^TA^T(B^T)^T = B^TA^TB$
$= B^TAB$ iff $A$ is symmetric
$\therefore$ $B^TAB$ is symmetric iff $A$ is symmetric
Also, $(B^TAB)^T = B^TA^TB = (-B)A^TB$
$\therefore$ $B^TAB$ is not skew symmetric if $B$ is skew symmetric.

**17. (d)** : $2ABC$ is not defined.
$\therefore$ There is no solution.

**18. (d)** : $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$

$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$

If $AB = BA$, then $a + 2c = a + 3b \Rightarrow 2c = 3b \Rightarrow b \neq 0$
Now, $b + 2d = 2a + 4b \Rightarrow 2a - 2d = -3b$

$$\therefore \quad \frac{a-d}{3b-c} = \frac{-\frac{3}{2}b}{3b-\frac{3}{2}b} = -1$$

**19. (b)** : $(f(\alpha))^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-\alpha)$

**20. (c)** : Given that $f(x) = \begin{cases} x^2 \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x^2 \text{ if } x < 0 \end{cases}$

**21. (d)** : $f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right) \in R$

$\Leftrightarrow -1 \le \log_2\left(\frac{x^2}{2}\right) \le 1 \Leftrightarrow \frac{1}{2} \le \frac{x^2}{2} \le 2$

$\Leftrightarrow 1 \le x^2 \le 4 \Leftrightarrow x \in [-2, -1] \cup [1, 2]$

**22. (a)** : $-1 \le \sin 3x \le 1$

**23. (a)** : Let $y = \frac{x^2 - x + 1}{x^2 + x + 1}$

$\Rightarrow yx^2 + yx + y = x^2 - x + 1$
$\Rightarrow (y - 1)x^2 + (y + 1)x + (y - 1) = 0$
Now, $x \in R \Rightarrow$ Discriminant $\ge 0$
$\Rightarrow (y + 1)^2 - 4(y - 1)^2 \ge 0 \Rightarrow -3y^2 + 10y - 3 \ge 0$
$\Rightarrow 3y^2 - 10y + 3 \le 0 \Rightarrow (3y - 1)(y - 3) \le 0 \Rightarrow \frac{1}{3} \le y \le 3$

$\therefore$ Range $= \left[\frac{1}{3}, 3\right]$

**24. (b)** : $5|x| - x^2 - 6 \ge 0 \Rightarrow x^2 - 5|x| + 6 \le 0$
When $x < 0$, $x^2 + 5x + 6 \le 0$, $-3 \le x \le -2$
When $x > 0$, $x^2 - 5x + 6 \le 0$, $2 \le x \le 3$
$x = 0$ will not satisfy the condition.
Domain is $[-3, -2] \cup [2, 3]$.

**25. (a)** : Here, $f(x) = x^2 + 1 + \frac{1}{x^2+1} - 1$

$\because \quad x^2 + 1 + \frac{1}{x^2+1} \ge 2 \Rightarrow x^2 + \frac{1}{x^2+1} \ge 1$

$\therefore$ $f(x) \in [1, \infty)$

**26. (b)** : Let $y = f(x) = (5 - (x - 8)^5)^{1/3}$
Then $y^3 = 5 - (x - 8)^5 \Rightarrow (x - 8)^5 = 5 - y^3$
$\Rightarrow x = 8 + (5 - y^3)^{1/5}$
Let, $z = g(x) = 8 + (5 - x^3)^{1/5}$

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Now, $f(g(x)) = [5 - (5 - x^3)^{1/5}]^5)^{1/3} = (5 - 5 + x^3)^{1/3} = x$

Similarly, we can show that $g(f(x)) = x$.

Hence, $g(x) = 8 + (5 - x^3)^{1/5}$ is the inverse of $f(x)$.

**27. (b) :** For function $f(x)$ to be defined we have $x^2 - 5x + 10 > 0$ ...(i) and $1 - \log_{10}(x^2 - 5x + 10) > 0$ ...(ii)

Now, (ii) $\Rightarrow \log_{10}(x^2 - 5x + 10) < 1 \Rightarrow x^2 - 5x + 10 < 10$

$\Rightarrow\ x^2 - 5x < 0 \Rightarrow x(x - 5) < 0 \Rightarrow 0 < x < 5$ ...(A)

Again, $x^2 - 5x + 10 > 0$ for all $x$, ...(B)

Since the discriminant of the corresponding equation $x^2 - 5x + 10 = 0$ is negative, so that the roots of the equation are imaginary.

Combining (A) and (B), we find that the domain of $f(x)$ is $(0, 5)$.

**28. (d) :** It is given that $2^x + 2^y = 2\ \forall\ x, y \in R$

Therefore, $2^x = 2 - 2^y < 2 \Rightarrow 0 < 2^x < 2$

Taking log for both side with base 2.

$\Rightarrow\ \log_2 0 < \log_2 2^x < \log_2 2$

Hence domain is $-\infty < x < 1$

**29. (c)**

**30. (b) :** $f(x) = x - \frac{1}{x}, \Rightarrow f(f(x)) = \frac{x^4 - 3x^2 + 1}{x(x^2 - 1)}$

Since, we have $f(f(f(x))) = 1 \Rightarrow f(f(x)) = f^{-1}(1) = \frac{1 + \sqrt{5}}{2}$

$\Rightarrow$ 2 values exist

or $f^{-1}(1) = \frac{1 - \sqrt{5}}{2} \Rightarrow$ 2 values exist

**31. (c) :** We have, $f(x) = 2\sin\left(x - \frac{\pi}{6}\right) + 2$

$\because$ $f$ is one–one and onto $\therefore f$ is invertible

Now, $fof^{-1}(x) = x \Rightarrow 2\sin\left(f^{-1}(x) - \frac{\pi}{6}\right) + 2 = x$

$$f^{-1}(x) = \sin^{-1}\left(\frac{x}{2} - 1\right) + \frac{\pi}{6} \quad \left(\therefore \left|\frac{x}{2} - 1\right| \le 1 \forall x \in [0, 4]\right)$$

Also, $\sin^{-1}\alpha + \cos^{-1}\alpha = \frac{\pi}{2}$

$$\Rightarrow\ f^{-1}(x) = \frac{\pi}{2} - \cos^{-1}\frac{x - 2}{2} + \frac{\pi}{6} = \frac{2\pi}{3} - \cos^{-1}\left(\frac{x - 2}{2}\right)$$

**32. (b) :** Let $y = f(x) = 1 + \alpha x \Rightarrow x = \frac{y - 1}{\alpha}$

$\Rightarrow f^{-1}(x) = \frac{x - 1}{\alpha}$

Now, $f(x) = f^{-1}(x) \Rightarrow \frac{x - 1}{\alpha} = 1 + \alpha x \Rightarrow x - 1 = \alpha + \alpha^2 x$

Equating the coefficient of $x$, we get

$\alpha^2 = 1$ and $\alpha = -1 \Rightarrow \alpha = -1$

**33. (c) :** We have, $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$

$$\Rightarrow\ 4f(x^2) + 6f\left(\frac{1}{x^2}\right) = 2x^2 - 2 \quad \text{...(i)}$$

Also, $2f\left(\frac{1}{x^2}\right) + 3f(x^2) = \frac{1}{x^2} - 1$

$$\Rightarrow\ 9f(x^2) + 6f\left(\frac{1}{x^2}\right) = \frac{3}{x^2} - 3 \quad \text{...(ii)}$$

$$\text{(ii)} - \text{(i)} \Rightarrow 5f(x^2) = \frac{3}{x^2} - 2x^2 - 1 \Rightarrow f(x^2) = \frac{3 - 2x^4 - x^2}{5x^2}$$

**34. (c) :** Taking $x = y = 1$, we get

$f(1)f(1) - f(1) = 2 \Rightarrow f^2(1) - f(1) - 2 = 0$

$\Rightarrow (f(1) - 2)(f(1) + 1) = 0 \Rightarrow f(1) = 2$ (as $f(1) > 0$)

Taking $y = 1$, we get $f(x) \cdot f(1) - f(x) = x + 1$

$\Rightarrow f(x) = x + 1 \Rightarrow f^{-1}(x) = x - 1$

$\therefore\ f(x) \cdot f^{-1}(x) = x^2 - 1$

**35. (d) :** Since $AB = B$ and $BA = A$

$\therefore$ $A$ and $B$ both are idempotent.

$(A - B)^2 = A^2 - AB - BA + B^2 = A - B - A + B = 0$

$\therefore$ $A - B$ is nilpotent.

**36. (15) :** $\sec x = 3 \Rightarrow \cos x = \frac{1}{3}$, which gives two values of $x$ in each of $[0, 2\pi]$, $(2\pi, 4\pi]$, $(4\pi, 6\pi]$ and one value in $6\pi + \frac{3\pi}{2} = \frac{15\pi}{2}$

$\therefore$ Greatest value of $n = 15$

**37. (5) :** $\sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\therefore\ K = 5$

**38. (6) :** $16(\sin^5 x + \cos^5 x) - 11(\sin x + \cos x) = 0$

$\Rightarrow\ (\sin x + \cos x)\{16(\sin^4 x - \sin^3 x \cos x + \sin^2 x \cos^2 x - \sin x \cos^3 x + \cos^4 x) - 11\} = 0$

$\Rightarrow\ (\sin x + \cos x)\{16(1 - \sin^2 x \cos^2 x - \sin x \cos x) - 11\} = 0$

$\Rightarrow\ (\sin x + \cos x)(4\sin x \cos x - 1)(4\sin x \cos x + 5) = 0$

As $4\sin x \cos x + 5 \neq 0$, so we have

$\sin x + \cos x = 0$ or $4\sin x \cos x - 1 = 0$

The required values are $\pi/12$, $5\pi/12$, $9\pi/12$, $13\pi/12$, $17\pi/12$, $21\pi/12$. Thus, there are 6 solutions on $[0, 2\pi]$.

**39. (2) :** Given equation is $\frac{1 + \sin x}{\cos x} = 2\cos x$

$\Rightarrow\ 1 + \sin x = 2\cos^2 x = 2(1 - \sin^2 x)$

$\Rightarrow\ 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow\ (1 + \sin x)(2\sin x - 1) = 0 \Rightarrow \sin x = -1$ or $1/2$

Now, $\sin x = -1 \Rightarrow \tan x$ and $\sec x$ is not defined.

$\sin x = 1/2 \Rightarrow x = \pi/6$ or $5\pi/6$.

$\therefore$ The required number of solutions is 2.

**40. (4) :** Given, $\log_{0.5}\sin x = 1 - \log_{0.5}\cos x,\ x \in [-2\pi, 2\pi]$
$\sin x > 0$ and $\cos x > 0$

$$\sin x \cos x = \frac{1}{2}$$

$\sin 2x = 1,\ 2x \in [-4\pi, 4\pi]$

$\Rightarrow$ 4 solutions

**41. (0.75) :** $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$

$$= \frac{1}{2}\left[1 + \cos 20° - (\cos 60° + \cos 40°) + (1 + \cos 100°)\right]$$

$$= \frac{1}{2}\left[1 + \cos 20° - \frac{1}{2} - \cos 40° + 1 - \cos 80°\right]$$

$$= \frac{1}{2}\left[\frac{3}{2} + \cos 20° - (2\cos 60° \cos 20°)\right] = \frac{3}{4} = 0.75$$

**42. (3) :** Given equation is $\sec^2(a + 2)x + a^2 - 1 = 0$
$\Rightarrow\ \tan^2(a + 2)x + a^2 = 0 \Rightarrow \tan^2(a + 2)x = 0$ and $a = 0$

$$\Rightarrow \tan^2 2x = 0 \Rightarrow x = 0, \frac{\pi}{2}, \frac{-\pi}{2}$$

$\therefore$ $(0, 0), (0, \pi/2), (0, -\pi/2)$ are ordered pairs satisfying the equation.

**43. (1155) :** $\dfrac{s-a}{11} = \dfrac{s-b}{12} = \dfrac{s-c}{13} = \dfrac{s}{36}$

On solving we get, $\tan^2(A/2) = \dfrac{13}{33} \Rightarrow \lambda = 1155$

**44. (1) :** Given, $2\sin x + 2\sin^2 x - 1 = 1$
or $\sin^2 x + \sin x - 1 = 0$

$$\therefore\ \sin x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1+\sqrt{5}}{2}$$

$$\Rightarrow \sin^2 x = \frac{3-\sqrt{5}}{2} \Rightarrow \cos^2 x = \frac{\sqrt{5}-1}{2}$$

$$\therefore\ \cos^2 x(1 + \cos^2 x) = \frac{\sqrt{5}-1}{2} \times \frac{\sqrt{5}+1}{2} = 1$$

**45. (4)**

**46. (6) :** Since $P^2 = I - P$ (Given) ...(i)
$\Rightarrow\ P^3 = P(I - P)$
$= P - P^2 = P - (I - P)$ (using(i))
$\therefore\ P^3 = 2P - I$ ...(ii)
Similarly, $P^4 = 2P^2 - P = 2I - 3P$ and $P^5 = 5P - 3I$ and $P^6 = 5P^2 - 3P = 5I - 8P$

**47. (144) :** $100A + 80 + 8 = 72\lambda_1$
$600 + 10B + 8 = 72\lambda_2$
$800 + 60 + C = 72\lambda_3,\ \lambda_1, \lambda_2, \lambda_3 \in I$

$$\text{Let } \Delta = \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$$

$$= \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 72\lambda_1 & 72\lambda_2 & 72\lambda_3 \end{vmatrix} (R_3 \leftrightarrow R_3 + 10R_2 + 100R_1) \quad \text{...(i)}$$

Now, $A88$ is divisible by $72 \Rightarrow A88$ is divisible by 9
$\therefore\ A = 2$
Also, $6B8$ is divisible by $9 \Rightarrow B = 4$
Substituting these values in (i) we get $\Delta$ is divisible by 144

**48. (6) :** $|A^{1003} - 5A^{1002}| = |A^{1002}(A - 5I)|$

$$= |A^{1002}|\,|A - 5I| = |A|^{1002}\,|A - 5I| = 1 \times \begin{vmatrix} 0 & -6 \\ 1 & -6 \end{vmatrix} = 6$$

**49. (2) :** Given: $f(x) = (-1)^{[x]}$.
First of all, we sketch the graph of $f(x)$ with the help of piecewise defined functions as follows:

$$f(x) = (-1)^{[x]} = \begin{cases} 1; & -2 \le x < -1 \\ -1; & -1 \le x < 0 \\ 1; & 0 \le x < 1 \\ -1: & 1 \le x < 2 \\ 1; & 2 \le x < 3. \end{cases}$$

$\because$ The function $f(x)$ repeats its value after the least interval of 2.
Therefore, the function $f(x)$ is periodic with period 2.

**50. (10) :** We have, $g(x)\,g(y) = g(x) + g(y) + g(xy) - 2$ ...(i)
Putting $x = 1$, $y = 2$ in (i), we have
$g(1)\,g(2) = g(1) + g(2) + g(2) - 2$
$\Rightarrow 5g(1) = 8 + g(1)\ \therefore\ \ g(1) = 2$

Also, replacing $y$ by $\dfrac{1}{x}$ in (i), we get

$$g(x)g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right) + g(1) - 2$$

$$\Rightarrow\ g(x)g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right)$$

$\Rightarrow\ g(x) = 1 \pm x^n$ ...(ii)
Put $x = 2$ in (ii), we get $\pm 2^n = 2^2$
Taking +ve sign, we set $n = 2$
$\therefore\ \ g(x) = 1 + x^2 \Rightarrow g(3) = 1 + 3^2 = 10$

❖❖

## SOLUTION SENDERS

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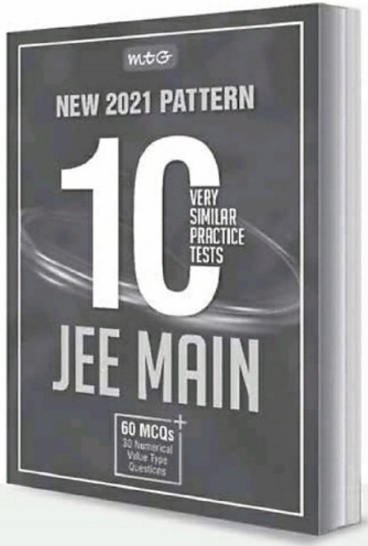
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# JEE ADVANCED

## PRACTICE PAPER

Exam on 3rd July

2021
PROBLEM

### SINGLE OPTION CORRECT TYPE

**1.** $f:\left(0,\ \frac{\pi}{2}\right)\to A,\ f(x)=\log_e(\sin x^{\sin x}+1)$, then the minimum value of $f(x)$ is

(a) $\log_e 2$ (b) $\log_e\left(\left(\frac{1}{e}\right)^{1/e}+1\right)$

(c) $\log_e((e)^2+1$ (d) 2

**2.** If $x^5-x^3+x=a$, then minimum value of $x^6$ is

(a) $a$ (b) $2a-1$ (c) $a+2$ (d) $a-3$

**3.** If $z$ be a complex number satisfying, $|z|^2+2(z+\bar{z})+3i(z-\bar{z})+4=0$ then complex number $z+3+2i$ will lie on

(a) circle with centre $1-5i$ and radius 4
(b) circle with centre $1+5i$ and radius 4
(c) circle with centre $1+5i$ and radius 3
(d) circle with centre $1-5i$ and radius 3

**4.** If $f(x-y)=f(x)g(y)-f(y)g(x)\ \forall\ x, y\in R$, where $f(x)$ is not identically zero, then

(a) If $f'(0^+)$ exists then $f'(0^-)$ also exists but not equal
(b) If $f'(0^+)$ exists then $f'(0^-)$ does not exist
(c) If $f'(0^+)$ exists then it is equal to $f'(0^-)$
(d) none of these

**5.** A quadratic polynomial maps from $[-2, 3]$ onto $[0, 3]$ and touches $x$-axis at $x=3$, then the polynomial is

(a) $\frac{3}{16}(x^2-6x+16)$ (b) $\frac{3}{25}(x^2-6x+9)$

(c) $\frac{3}{25}(x^2-6x+16)$ (d) none of these

**6.** Let $f(x)=\int_0^x(t^2+2t+2)\,dt$, where $x$ belongs to the set of real numbers satisfying the inequation $\log_2(1+\sqrt{6x-x^2-8})\ge 0$. If $f(x)\in[\alpha,\ \beta]$, then maximum value of $|\alpha-\beta|$ is

(a) $\frac{103}{3}$ (b) $\frac{104}{3}$

(c) $\frac{106}{3}$ (d) none of these

**7.** The set of values of $a$ for which inequation $(a-1)x^2-(a+1)x+a-1\ge 0$ is true for all $x\ge 2$ is

(a) $\left(1,\ \frac{7}{3}\right]$ (b) $(-\infty, 1)$

(c) $\left[\frac{7}{3},\ \infty\right)$ (d) none of these

**8.** If $x_1, x_2, x_3, ..., x_n$ are the roots of $x^n+ax+b=0$, then the value of $(x_1-x_2)(x_1-x_3)(x_1-x_4)...(x_1-x_n)$ is

(a) $n_1^n$ (b) $nx_1^{n-1}+a$

(c) $nx_1$ (d) $nx_1+a^n$

**9.** The value of $\lim_{n\to\infty}\left(\frac{n!}{(mn)^n}\right)^{1/n}$ is

(a) $em$ (b) $\frac{e}{m}$

(c) $\frac{1}{em}$ (d) none of these

**10.** If one of the roots of the equation $\begin{vmatrix}7 & 6 & x^2-13\\ 2 & x^2-13 & 2\\ x^2-13 & 3 & 7\end{vmatrix}=0$ is $x=2$, then sum of all other five roots is

(a) $2\sqrt{15}$ (b) $-2$

(c) $\sqrt{20}+\sqrt{15}-2$ (d) none of these

**11.** A unit vector is orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar to $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$, then the vector is

---

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(a) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (b) $\frac{2\hat{i}+5\hat{j}}{\sqrt{29}}$

(c) $\frac{6\hat{i}-5\hat{k}}{\sqrt{61}}$ (d) $\frac{2\hat{i}+2\hat{j}-\hat{k}}{3}$

**12.** The range of the function $y=\sqrt{2\{x\}-\{x\}^2-\frac{3}{4}}$ is (where $\{\cdot\}$ denotes fractional part)

(a) $\left[-\frac{1}{4}, \frac{1}{4}\right]$ (b) $\left[0, \frac{1}{2}\right)$

(c) $\left[0, \frac{1}{4}\right]$ (d) $\left[\frac{1}{4}, \frac{1}{2}\right]$

**13.** $\int \frac{dx}{\sqrt{\cos^3 x\cos(x-\alpha)}}$ is equal to

(a) $2\sqrt{\tan x\sin\alpha+\cot\alpha}+c$

(b) $2\operatorname{cosec}\alpha\sqrt{\tan x\sin\alpha+\cos\alpha}+c$

(c) $2\sqrt{\sin\alpha+\tan x\cos\alpha}+c$

(d) $2\operatorname{cosec}\alpha\sqrt{\tan x\cos\alpha+\sin\alpha}+c$

**14.** Inside a big circle exactly $n$ small circles each of radius $r$ can be drawn in such a way that each small circle touches the big circle and two small circles. If $n \geq 3$, then the radius of the bigger circle is

(a) $r\operatorname{cosec}\left(\frac{\pi}{n}\right)$ (b) $r\left\{1+\operatorname{cosec}\left(\frac{\pi}{n}\right)\right\}$

(c) $r\left\{1+\operatorname{cosec}\left(\frac{2\pi}{n}\right)\right\}$ (d) $r\left\{1+\operatorname{cosec}\left(\frac{\pi}{2n}\right)\right\}$

**15.** If $m_1$ and $m_2$ be the slopes of two perpendicular chords of equal length passing through origin of circle $(x-1)^2+(y+2)^2=5$, then the value of $m_1^2+m_2^2$ is equal to

(a) $\frac{80}{9}$ (b) $\frac{82}{9}$

(c) $\frac{83}{9}$ (d) none of these

**16.** If the system of equations $x-ky-z=0$, $kx-y-z=0$, $x+y-z=0$ has a non-zero solutions, then the possible values of $k$ are

(a) $-1, 2$ (b) $0, 1$ (c) $1, 2$ (d) $-1, 1$

**17.** Period of $|\cos|x|| + |\sin|x||$ is

(a) $2\pi$ (b) $\pi$

(c) $\pi/2$ (d) none of these

**18.** If $O$ is the origin and $A$ and $B$ are respectively $(a_1, b_1)$ and $(a_2, b_2)$, then $OA \cdot OB \sin(AOB)$ is equal to

(a) $a_1a_2 + b_1b_2$ (b) $a_1b_2 + a_2b_1$

(c) $b_1a_2 - b_2a_1$ (d) $a_1a_2 - b_1b_2$

**19.** If $f(x)=[|x|]$, then $\int_0^{100} f(x)dx$ is equal to (where $[\cdot]$ denotes the greatest integer function)

(a) 4950 (b) 5000 (c) 4250 (d) 4590

**20.** The largest term of $a_n=\frac{n^2}{n^3+200}$, $n \in N$ is

(a) $\frac{29}{453}$ (b) $\frac{49}{543}$ (c) $\frac{43}{553}$ (d) $\frac{41}{451}$

**21.** If $\vec{r}=\lambda(\vec{a}\times\vec{b})+\mu(\vec{b}\times\vec{c})+\gamma(\vec{c}\times\vec{a})$ and $[\vec{a}\,\vec{b}\,\vec{c}]=\frac{1}{8}$, then $\lambda+\mu+\gamma$ is

(a) $8(\vec{r}.\vec{a})$ (b) $8(\vec{r}.\vec{b})$

(c) $8(\vec{r}.\vec{c})$ (d) $8\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$

## ONE OR MORE THAN ONE OPTION CORRECT TYPE

**22.** If $[x]$ represents the greatest integer less than or equal to $x$, then which of the following statement is true?

(a) $\sin[x] = \cos[x]$ has no solution

(b) $\sin[x] = \tan[x]$ has infinitely many solutions

(c) $\sin[x] = \cos[x]$ possess unique solution

(d) $\sin[x] = \tan[x]$ for no value of $x$

**23.** If $p$, $q$, $r$ are in H.P. and $p$, $q$, $-2r$ are in G.P; $(p, q, r > 0)$ then

(a) $p^2, q^2, r^2$ are in G.P. (b) $p^2, q^2, r^2$ are in A.P.

(c) $2p, q, 2r$ are in A.P. (d) $p+q+r=0$

**24.** The det $\Delta=\begin{vmatrix} d^2+r & de & df \\ de & e^2+r & ef \\ df & ef & f^2+r \end{vmatrix}$ is divisible by

(a) $r$ (b) $(d+e^2+f^2+r)$

(c) $(d^2+e^2+f^2+r)$ (d) $(d^2+e+f^2+r^2)$

**25.** If both the roots of equation $ax^2+x+c-a=0$ are imaginary and $c > -1$, then

(a) $3a > 2+4c$ (b) $a > 0$

(c) $c < a$ (d) None of these

**26.** The solution of the equation

$$3^{\sin 2x+2\cos^2 x}+3^{1-\sin 2x+2\sin^2 x}=28$$

(a) $(2n+1)\frac{\pi}{2}, n\in I$ (b) $n\pi-\frac{\pi}{4}, n\in I$

(c) $2n\pi$ (d) none of these

**27.** If $\sin y = x\cos(a+y)$, then $\frac{dy}{dx}$ is

(a) $\frac{\cos^2(a+y)}{\cos a}$ (b) $\frac{\cos(a+y)}{x\sin(a+y)+\cos y}$

(c) $\frac{\cos a}{3x^2-2x\cos a-1}$ (d) none of these

### COMPREHENSION TYPE

**Passage for Q.No. 28 to 30**

A ray of light is coming along the line $L = 0$ and strikes the plane mirror kept along the plane $P = 0$ at $B$. $A(2, 1, 6)$ is a point on the line $L = 0$ whose image about $P = 0$ is $A'$. It is given that $L = 0$ is $\frac{x-2}{3}=\frac{y-1}{4}=\frac{z-6}{5}$ and $P = 0$ is $x + y - 2z = 3$.

**28.** Co-ordinates of $A'$ are

(a) $(6, 5, 2)$ (b) $(6, 5, -2)$

(c) $(6, -5, -2)$ (d) none of these

**29.** Co-ordinates of $B$ are

(a) $(5, 10, 6)$ (b) $(10, 15, 11)$

(c) $(-10, -15, -14)$ (d) none of these

**30.** If $L_1 = 0$ is the reflected ray, then its equation is

(a) $\frac{x+10}{4}=\frac{y-5}{4}=\frac{z+2}{3}$

(b) $\frac{x+10}{3}=\frac{y+15}{5}=\frac{z+14}{5}$

(c) $\frac{x+10}{4}=\frac{y+15}{5}=\frac{z+14}{3}$

(d) none of these

**Passage for Q.No. 31 to 33**

Given $A(3 + 4i)$, $B(-4 + 3i)$ and $C(4 + 3i)$ be the vertices of $\Delta ABC$ which is inscribed in a circle $S = 0$. Let $AD$, $BE$, $CF$ are altitudes through $A$, $B$, $C$ which meet the circle $S = 0$ at $D(z_1)$, $E(z_2)$ and $F(z_3)$ respectively.

**31.** Complex number $z_1$ is equal to

(a) $(2\sqrt{6}+i)$ (b) $(1+2\sqrt{6}i)$

(c) $3 - 4i$ (d) $-3 - 4i$

**32.** Orthocentre of triangle $ABC$ is at

(a) $2 + 9i$ (b) $3 + 10i$

(c) $3 + 11i$ (d) none of these

**33.** The value of $z_1z_2z_3$ is equal to

(a) $75 + 100i$ (b) $-100 +75i$

(c) $100 + 75i$ (d) none of these

**Passage for Q.No. 34 to 36**

Consider an ellipse $\frac{x^2}{4}+y^2=\alpha$, ($\alpha$ is parameter > 0) and a parabola $y^2 = 8x$. A common tangent to the ellipse and the parabola meets the co-ordinate axes at $A$ & $B$.

**34.** Locus of mid point of $AB$ is

(a) $y^2 = -2x$ (b) $y^2 = -x$

(c) $y^2=-\frac{x}{2}$ (d) $\frac{x^2}{4}+\frac{y^2}{2}=1$

**35.** If the eccentric angle of a point on the ellipse where the common tangent meets it is $\left(\frac{2\pi}{3}\right)$, then $\alpha$ is equal to

(a) 4 (b) 5 (c) 26 (d) 36

**36.** If two of the three normals drawn from the point $(h, 0)$ on the ellipse to the parabola $y^2 = 8x$ are perpendicular, then

(a) $h = 2$ (b) $h = 3$ (c) $h = 4$ (d) $h = 6$

### MATRIX MATCH TYPE

**37.** Match the following :

| | List-I | | List-II |
|---|---|---|---|
| (A) | If $x, y, z \in N$, then number of ordered triplet $(x, y, z)$ satisfying $xyz = 243$ is | (1) | 19 |
| (B) | The number of terms in the expansion of $(x + y + z)^6$ is | (2) | 28 |
| (C) | If $x \in N$, then number of solutions of $x^2 + x - 400 \le 0$ is | (3) | 21 |
| (D) | If $x, y, z \in N$, then number of solution of $x + y + z = 10$ | (4) | 36 |

**38.** Match the following :

| | List-I | | List-II |
|---|---|---|---|
| (A) | If inequation $x^2 - ax + a > 0\ \forall\ x \in R$, then $a$ belongs to | (1) | $[0, 4)$ |
| (B) | If $x^3-3x+\frac{a}{2}=0$ has three real and distinct root, then $\lvert a\rvert$ belong to | (2) | $[0, 3]$ |
| (C) | If $x^3 + ax^2 + x + 1 = 0$ is an increasing function, then $a^2$ may belong to | (3) | $(0, 4)$ |
| (D) | If quadratic equation $x^2 + 3ax + a^2 - 9 = 0$ has roots of opposite sign, then $a$ belongs to | (4) | $(-3, 3)$ |

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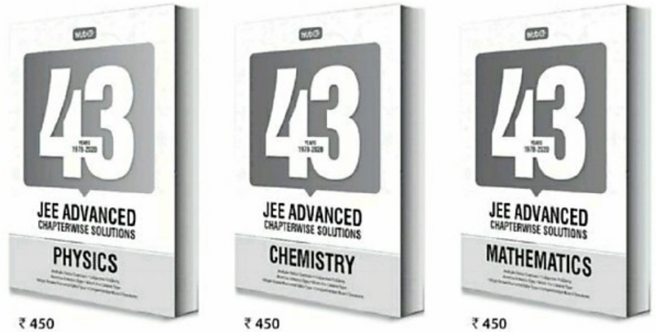
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**39.** Match the following :

| | List-I | | List-II |
|---|---|---|---|
| (A) | The set of values of $b$ for which the function $f(x) = x^2 + bx + 5$ is an increasing function on $[1, 3]$ is | (1) | $(-\infty, 0)$ |
| (B) | India plays 2 matches with Sri Lanka and South Africa each. In any match the probability of India getting points 0, 1, 2 are 0.15, 0.35, 0.5 respectively assuming the outcomes are independent. The probability of getting at least 7 points is $p$, where $p$ belongs to | (2) | $\left(\frac{1}{10}, \infty\right)$ |
| (C) | The domain of the function $f(x)=\frac{1}{\sqrt{\lvert x\rvert - x}}$ is | (3) | $[-2, \infty)$ |

## NUMERICAL VALUE TYPE

**40.** In $\Delta ABC$ if $BC$ is unity,

$$\sin\frac{A}{2}=x_1,\ \sin\frac{B}{2}=x_2,\ \cos\frac{A}{2}=x_3 \text{ and } \cos\frac{B}{2}=x_4$$

with $\left(\frac{x_1}{x_2}\right)^{2007}-\left(\frac{x_3}{x_4}\right)^{2006}=0$, then the length of $AC$ is ________.

**41.** If $l_n=\int_0^\infty e^{-x}(\sin x)^n\,dx\,(n>1)$, then the value of $\frac{101 l_{10}}{90 l_8}$ is equal to ________.

**42.** The A.M., G.M. and H.M. of first and last term of the series 100, 101, 102,...., $n$ are also the term of this series, then the value of $\frac{n}{200}\,(100 < n \le 500)$ is ________.

## SOLUTIONS

**1. (b) :** Let $h(x) = \log(x^x + 1)$, for $0 < x < 1$. Then, $h'(x) > 0$ if $x > \frac{1}{e}$. Thus, for $\frac{1}{e} < x < 1$, function will be increasing. Also, $h'(x) < 0$, if $x < \frac{1}{e}$. So, at $x = \frac{1}{e}$, there is minimum. $f_{\min} = \log_e\left(\left(\frac{1}{e}\right)^{1/e} + 1\right)$.

**2. (b) :** $x(x^4 - x^2 + 1) = \frac{x(x^6+1)}{x^2+1}$. Suppose $x > 0$, then

$a > 0$ and $x^6 + 1 = \frac{a(x^2+1)}{x} = a\left(x+\frac{1}{x}\right) \ge 2a,\ x^6 > 2a - 1.$

If $x \le 0 \Rightarrow a \le 0,\ 2a - 1 < 0 \le x^6 \Rightarrow x^6 > 2a - 1.$

**3. (c) :** Given equation is

$|z|^2 + z(2 + 3i) + \bar{z}(2 - 3i) + 4 = 0$, which represents a circle with centre $-(2 - 3i)$, radius $= \sqrt{a\bar{a} - b} = 3$

Let $\omega = z + 3 + 2i = z + 2 - 3i + 1 + 5i$,

$|\omega - 1 - 5i| = |z + 2 - 3i| = 3$

so, $\omega$ lies on circle whose centre is $1 + 5i$, radius $= 3$.

**4. (c) :** $f(x - y) = f(x)g(y) - f(y)g(x)$ ...(i)

Put $x = y$ in (i), $f(0) = 0$,

Put $y = 0$ in (i), $f(x) = f(x)\,g(0) - f(0)\,g(x) \Rightarrow g(0) = 1$

## SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 3 | | 8 | | 9 | 6 | 4 | 2 | | | | 3 | 4 | | 9 | 5 | 8 | 2 | | |
| | 8 | | | 3 | | 7 | 1 | | | | | | | 6 | | 7 | 4 | | 3 | |
| 4 | 1 | | 7 | 6 | | | | 8 | | | | | 8 | | 1 | 3 | | 5 | 7 | |
| 2 | 7 | | 5 | | 1 | | | 3 | | | | | | | | 9 | 2 | 7 | 8 | 6 |
| 3 | 5 | 8 | 9 | 4 | | | 7 | 1 | | | | 7 | | 2 | | 4 | 3 | | 9 | |
| 6 | 4 | 1 | | | 2 | | 8 | 9 | | | | | 9 | | 7 | | 1 | | | |
| | | | | 9 | | 3 | 2 | 7 | | | 8 | 9 | 5 | 1 | 3 | 8 | | | 4 | 2 |
| 1 | | | | 2 | | | 5 | | | | | 6 | 7 | | 4 | 2 | | | | 9 |
| | | | | | 3 | 1 | 9 | 6 | | | 3 | 4 | 2 | | 6 | | 9 | 3 | 5 | |
| | | | | | | | 6 | 9 | 3 | | 4 | 8 | 1 | | | | | | | |
| | | | | | | | | 1 | 9 | 6 | 5 | | | | | | | | | |
| | | | | | | 7 | | | 8 | 1 | | 5 | 6 | | | | | | | |
| | 8 | | | 1 | | 6 | | | | 8 | 7 | | 9 | | | | | 5 | | |
| 9 | | 3 | | | | 4 | | | | | 6 | 7 | 3 | | | 8 | 2 | | 6 | |
| | 1 | 6 | | | | 9 | 7 | 5 | | | 1 | | 8 | | 9 | | 1 | 4 | | 3 |
| 3 | 7 | | 1 | | 4 | | 2 | 6 | | | | 5 | 4 | | 6 | 2 | | | 1 | 7 |
| | 4 | 2 | | 7 | | 1 | | | | | | 6 | 7 | | | | 8 | | 4 | 2 |
| | 9 | | | 6 | 2 | 8 | 4 | | | | | | 2 | | 3 | | | | | 6 |
| | 6 | 4 | | 3 | 1 | | | 9 | | | | | | 8 | | | 7 | 6 | | |
| | 3 | | | | 8 | | 6 | | | | | | 6 | | 8 | | 5 | | 3 | 1 |
| 1 | 2 | 9 | 7 | | 6 | 3 | 8 | | | | | | 5 | 2 | 1 | 6 | 3 | 7 | | 4 |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.*

Now, $f'(0^+) = \lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$= \lim\limits_{h\to 0}\dfrac{f(0)g(-h)-f(-h)g(0)}{h} = \lim\limits_{h\to 0}\dfrac{f(-h)}{-h} = f'(0^-)$

*i.e.* $f'(0^+) = f'(0^-)$.

**5. (b) :** Let $f(x) = ax^2 + bx + c$.

As it touches $x$-axis at $x = 3$

$\Rightarrow \dfrac{-b}{2a} = 3 \Rightarrow b = -6a$ ...(i)

Also, $9a + 3b + c = 0$ ...(ii)

$4a - 2b + c = 3$ ...(iii)

Solving (i), (ii) and (iii), we get

$a = \dfrac{3}{25}, b = -\dfrac{18}{25}, c = \dfrac{27}{25} \Rightarrow f(x) = \dfrac{3}{25}(x^2 - 6x + 9)$

**6. (b) :** $\log_2(1+\sqrt{6x-x^2-8}) \ge 0$

$\Rightarrow 1+\sqrt{6x-x^2-8} \ge 1$

$\Rightarrow 6x - x^2 - 8 \ge 0 \Rightarrow x^2 - 6x + 8 \le 0$

$\Rightarrow (x-2)(x-4) \le 0 \Rightarrow 2 \le x \le 4$

Now $f'(x) = x^2 + 2x + 2 > 0 \ \forall\ x \in R$

$\Rightarrow f(x)$ is strictly increasing in $[2, 4]$

$f(x) = \dfrac{x^3}{3} + x^2 + 2x,\ \alpha = f(2) = \dfrac{8}{3} + 4 + 4 = \dfrac{32}{3}$

$\beta = f(4) = \dfrac{64}{3} + 16 + 8 = \dfrac{136}{3},\ |\alpha - \beta|_{\max} = \dfrac{104}{3}$

**7. (c) :** Given inequation is

$(a-1)x^2 - (a+1)x + a - 1 \ge 0$

$\Rightarrow a(x^2 - x + 1) - (x^2 + x + 1) \ge 0$

$\Rightarrow a \ge \dfrac{x^2+x+1}{x^2-x+1} = 1 + \dfrac{2x}{x^2-x+1} = 1 + \dfrac{2}{x+\frac{1}{x}-1}$ ...(i)

Let $y = x + \dfrac{1}{x}$, $y$ is increasing in $[2, \infty)$

$\Rightarrow 1 + \dfrac{2}{x+\frac{1}{x}-1} \in \left(1, \dfrac{7}{3}\right]$ for all $x \ge 2$.

Thus, given inequation will be true for all $x \ge 2$, if

$a \ge \dfrac{7}{3}$ [Using (i)]

**8. (b) :** Given $x^n + ax + b = 0 = (x - x_1)(x - x_2)(x - x_3) \ldots (x - x_n)$

$\Rightarrow (x-x_2)(x-x_3)\ldots(x-x_n) = \dfrac{x^n + ax + b}{x - x_1}$

$\therefore\ (x_1 - x_2)(x_1 - x_3)(x_1 - x_4) \ldots (x_1 - x_n)$

$= \lim\limits_{x\to x_1}\left(\dfrac{x^n+ax+b}{x-x_1}\right) = nx_1^{n-1} + a.$

**9. (c) :** Let $L = \lim\limits_{n\to\infty}\dfrac{1}{m}\left(\dfrac{1}{n}\cdot\dfrac{2}{n}\cdot\dfrac{3}{n}\ldots\dfrac{n}{n}\right)^{1/n}$

$\Rightarrow \ln L = \lim\limits_{n\to\infty}\left[\ln\left(\dfrac{1}{m}\right) + \dfrac{1}{n}\left(\ln\dfrac{1}{n} + \ln\dfrac{2}{n} + \ldots\ldots + \ln\dfrac{n}{n}\right)\right]$

$= -\ln m + \lim\limits_{n\to\infty}\dfrac{1}{n}\sum\limits_{r=1}^{n}\ln\left(\dfrac{r}{n}\right)$

$= -\ln m + \int\limits_0^1 \ln x\,dx = -\ln m - 1 = \ln\left(\dfrac{1}{em}\right)\ \therefore\ L = \dfrac{1}{em}$

**10. (b) :** Let $x^2 - 13 = t$, then $t^3 - 67t + 126 = 0$

$\Rightarrow t = -9, 2, 7 \Rightarrow x = \pm 2, \pm\sqrt{20}, \pm\sqrt{15}$.

**11. (a) :** A vector orthogonal to $\vec{c}$ and coplanar with $\vec{a}$ and $\vec{b}$ is $\vec{c}\times(\vec{a}\times\vec{b})$

Here, $\vec{c} = (5, 2, 6), \vec{a} = (2, 1, 1), \vec{b} = (1, -1, 1)$

Then $\vec{c}\times(\vec{a}\times\vec{b}) = (0, 27, -9)$

$\therefore$ Unit vector is $\dfrac{3\hat{j}-\hat{k}}{\sqrt{10}}$.

**12. (b) :** Set $\{x\} = t$ so that $0 \le t < 1$, $y = \sqrt{-t^2 + 2t - \dfrac{3}{4}}$

The domain of function is $\dfrac{1}{2} \le t < 1$.

$y\left(\dfrac{1}{2}\right) = \sqrt{-\dfrac{1}{4} + 1 - \dfrac{3}{4}} = 0;\ y(1) = \sqrt{-1 + 2 - \dfrac{3}{4}} = \sqrt{\dfrac{1}{4}} = \dfrac{1}{2}$

But $\dfrac{1}{2}$ is not attained, so the range is $\left[0, \dfrac{1}{2}\right)$.

**13. (b) :** Let $I = \displaystyle\int\dfrac{dx}{\sqrt{\cos^3 x(\cos x\cos\alpha + \sin x\sin\alpha)}}$

$= \displaystyle\int\dfrac{\sec^2 x\,dx}{\sqrt{\cos\alpha + \tan x\sin\alpha}}$, Put $\tan x\ \sin\alpha + \cos\alpha = t^2$

$\Rightarrow \sec^2 x\,dx = \dfrac{2t\,dt}{\sin\alpha}\ \therefore\ I = \displaystyle\int\dfrac{2t\,dt}{\sin\alpha\cdot t} = 2\operatorname{cosec}\alpha\cdot t$

$= 2\operatorname{cosec}\alpha\sqrt{\tan x\sin\alpha + \cos\alpha} + c$

**14. (b) :** We have, $\sin\left(\dfrac{\pi}{n}\right) = \dfrac{r}{R-r} \Rightarrow R - r = r\operatorname{cosec}\left(\dfrac{\pi}{n}\right)$

$\therefore\ R = r\left\{1 + \operatorname{cosec}\left(\dfrac{\pi}{n}\right)\right\}$

**15. (b) :** Let slope of chord, $OA = m$, $\tan 45° = \left|\dfrac{m+2}{1-2m}\right|$

$\Rightarrow |m+2| = |1-2m| \Rightarrow m + 2 = \pm(1-2m) \Rightarrow m = 3$ or $-\dfrac{1}{3}$

$$\Rightarrow m_1^2+m_2^2=(m_1+m_2)^2-2m_1m_2=\frac{64}{9}+2=\frac{82}{9}$$

**16. (d) :** Set of homogeneous equations will have non-trivial solutions if $\Delta = 0$

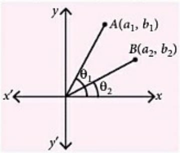
$$\Rightarrow \begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 1 & -k & 0 \\ k & -1 & k-1 \\ 1 & 1 & 0 \end{vmatrix}=0 \qquad [\text{Applying } C_3 \to C_3 + C_1]$$

$$\Rightarrow k^2-1=0 \Rightarrow k=\pm 1$$

**17. (c) :** $f(x)=f\left(\frac{\pi}{2}+x\right)$, where $f(x) = |\cos|x|| + |\sin|x||$.

**18. (c) :** $\sin(\theta_1-\theta_2)=\sin\theta_1\cos\theta_2-\cos\theta_1\sin\theta_2$



$$=\frac{b_1}{\sqrt{a_1^2+b_1^2}}\cdot\frac{a_2}{\sqrt{a_2^2+b_2^2}}-\frac{a_1}{\sqrt{a_1^2+b_1^2}}\cdot\frac{b_2}{\sqrt{a_2^2+b_2^2}}$$

$$\Rightarrow \sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}\sin(\theta_1-\theta_2)=b_1a_2-b_2a_1$$

$$\Rightarrow OA\cdot OB\sin(AOB)=b_1a_2-b_2a_1$$

**19. (a) :** $\int_0^1 0dx+\int_1^2 1dx+\int_2^3 2dx+\ldots+\int_{99}^{100} 99dx=\frac{99\times 100}{2}$

$$=4950$$

**20. (b) :** Let $f(x)=\frac{x^2}{x^3+200}, x\geq 1$

$$f'(x)=\frac{x(400-x^3)}{(x^3+200)^2}$$

$f'(x)>0$ if $1\leq x<\sqrt[3]{400}$ and $f'(x)<0$ if $x>\sqrt[3]{400}$

$\Rightarrow$ $f(x)$ has a maximum at $x=\sqrt[3]{400}, 7<\sqrt[3]{400}<8$

$\Rightarrow$ Either $a_7$ or $a_8$ is largest term.

Here, $a_7=\frac{49}{543}, a_8=\frac{8}{89}, a_7>a_8$

$\therefore$ Largest term $=\frac{49}{543}$

**21. (d) :** $[\bar{a}\bar{b}\bar{c}]=\bar{a}\cdot(\bar{b}\times\bar{c})=\bar{b}\cdot(\bar{c}\times\bar{a})=\bar{c}\cdot(\bar{a}\times\bar{b})$

$$=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=D$$

Given $\bar{r}=\lambda(\bar{a}\times\bar{b})+\mu(\bar{b}\times\bar{c})+\gamma(\bar{c}\times\bar{a}), [\bar{a}\bar{b}\bar{c}]=\frac{1}{8}$

Multiply the above relation with $\vec{a},\vec{b},\vec{c}$, we get

$\bar{r}\cdot\bar{a}=\lambda\cdot 0+\mu[\bar{a}\bar{b}\bar{c}]+0=\frac{\mu}{8}\Rightarrow 8(\bar{r}\cdot\bar{a})=\mu$

Similarly $\gamma=8(\vec{r}\cdot\vec{b}), \lambda=8(\vec{r}\cdot\vec{c})$

$\therefore\ \lambda+\mu+\gamma=8\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$

**22. (a, b) :** From option (a), we have $\sin[x]=\cos[x]$

$$\Rightarrow [x]=n\pi+\left(\frac{\pi}{4}\right)=\left(n+\frac{1}{4}\right)\pi,$$

Which is not possible as LHS $\in Z$ and RHS $\notin Z$.
Hence, $\sin[x]=\cos[x]$ has no solution.
However, $\sin[x]=\tan[x]\Rightarrow[x]=n\pi, n\in I$ which is possible when $[x]=0$ and $n=0$ $\therefore$ $0\leq x<1$.

**23. (b, c) :** $p, q, r$ are in H.P. $\Rightarrow q=\frac{2pr}{p+r}$

$\Rightarrow q^2(p+r)^2=(2pr)^2$

Also, $q^2=-2pr\Rightarrow q^2(p+r)^2=q^4$

$\Rightarrow q^2[(p+r)^2-q^2]=0\Rightarrow(p+r+q)(p+r-q)=0,$

$\Rightarrow p+r=q \qquad (\because p+q+r\neq 0)$

$\therefore$ $p, \frac{q}{2}, r$ are in A.P. or $2p, q, 2r$ are in A.P.

Now, $q^2[p^2+r^2+2pr]=q^4\Rightarrow q^2[p^2+r^2-q^2]=q^4$

$\Rightarrow [p^2+r^2-2q^2]=0$

or $p^2+r^2=2q^2$ $\therefore$ $p^2, q^2, r^2$ are in A.P.

**24. (a, c) :** Multiply $R_1, R_2, R_3$ by $d, e, f$ and take out $d, e, f$ common from $C_1, C_2, C_3$, then $\Delta=\begin{vmatrix} d^2+r & d^2 & d^2 \\ e^2 & e^2+r & e^2 \\ f^2 & f^2 & f^2+r \end{vmatrix}$

Operate $R_1\to R_1+R_2+R_3$ and take out $d^2+e^2+f^2+r$ common from $R_1$, then

$$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ e^2 & e^2+r & e^2 \\ f^2 & f^2 & f^2+r \end{vmatrix}(d^2+e^2+f^2+r)$$

$$=r^2(d^2+e^2+f^2+r).$$

**25. (b) :** Here $f(1)=a+1+c-a$

As $c>-1\ \Rightarrow\ f(1)>0$

**26. (a, b) :** We have,

$$3^{\sin 2x+2\cos^2 x}+3^{1-\sin 2x+2\sin^2 x}=28,\ y+\frac{27}{y}=28,$$

$\Rightarrow\ y^2 - 28y + 27 = 0 \Rightarrow y = 1$

($\because\ y \neq 27$, since max. $(\sin 2x + \cos 2x) = \sqrt{2}$ )

$\therefore\ \ \sin 2x + 2\cos^2 x = 0 \Rightarrow 2\cos x(\sin x + \cos x) = 0$

$\Rightarrow\ \ x=(2n+1)\dfrac{\pi}{2}$ or $x=n\pi-\dfrac{\pi}{4}$

**27. (a, b) :** $\sin y = x\cos(a+y) \Rightarrow \dfrac{\sin y}{\cos(a+y)} = x$ ...(i)

$$\frac{dx}{dy}=\frac{\cos y\cos(a+y)+\sin y\sin(a+y)}{\cos^2(a+y)}=\frac{\cos a}{\cos^2(a+y)}$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{\cos^2(a+y)}{\cos a} \quad \text{...(ii)}$$

$$=\frac{\cos^2(a+y)}{\cos y\cos(a+y)+\sin y\sin(a+y)}=\frac{\cos(a+y)}{\cos y+x\sin(a+y)}$$

[Using (i)]

**28. (b) :** Let $A'(x_2, y_2, z_2)$ be image of $A(2, 1, 6)$ about mirror $x + y - 2z = 3$, then

$$\frac{x_2-2}{1}=\frac{y_2-1}{1}=\frac{z_2-6}{-2}=\frac{-2(2+1-12-3)}{1^2+1^2+2^2}=4$$

$\Rightarrow\ (x_2, y_2, z_2) = (6, 5, -2)$

**29. (c) :** Let $\dfrac{x-2}{3}=\dfrac{y-1}{4}=\dfrac{z-6}{5}=\lambda,$

$\therefore\ \ x = 2 + 3\lambda,\ y = 1 + 4\lambda,\ z = 6 + 5\lambda$, lies on plane $x + y - 2z = 3$

$\therefore\ \ 2 + 3\lambda + 1 + 4\lambda - 2(6 + 5\lambda) = 3$

$\Rightarrow\ \ 3 + 7\lambda - 12 - 10\lambda = 3 \Rightarrow -3\lambda = 12\ \Rightarrow \lambda = -4$

$\therefore\ \ $ Point $B \equiv (-10, -15, -14)$.

**30. (c) :** Equation of reflected ray $L_1 = 0$ is a line joining $A'(6, 5, -2)$ and $B(-10, -15, -14)$

*i.e.* $\dfrac{x+10}{16}=\dfrac{y+15}{20}=\dfrac{z+14}{12} \Rightarrow \dfrac{x+10}{4}=\dfrac{y+15}{5}=\dfrac{z+14}{3}$

**31. (c) :** $z_1=-\dfrac{(-4+3i)(4+3i)}{3+4i}=3-4i$

**32. (b) :** Circumcentre of $\Delta ABC = (0, 0)$.

Now, centroid of $\Delta ABC=\dfrac{3+10i}{3}$

$\therefore\ \ $ Orthocentre of $\Delta ABC = 3 + 10i$.

**33. (a) :** $z_1z_2z_3 = -z_az_bz_c = -(3 + 4i)(-4 + 3i)(4 + 3i)$
$= (3 + 4i)(4 - 3i)(4 + 3i) = (3 + 4i)25 = 75 + 100i$

**(34-36) :** **34. (b)** **35. (d)** **36. (d)**

Equation of tangent to $y^2 = 8x$ is $yt - x - 2t^2 = 0$ ...(1)

Equation of tangent to ellipse is $\dfrac{x\cos\theta}{2\sqrt{\alpha}}+\dfrac{y\sin\theta}{\sqrt{\alpha}}=1$ ...(2)

If the tangent meets the coordinate axes at $A$ and $B$ then $A$ is $\left(\dfrac{2\sqrt{\alpha}}{\cos\theta},\ 0\right)$ and $B$ is $\left(0,\ \dfrac{\sqrt{\alpha}}{\sin\theta}\right)$.

Let mid point of $AB$ is $(h, k)$ then,

$$h=\frac{\sqrt{\alpha}}{\cos\theta},k=\frac{\sqrt{\alpha}}{2\sin\theta},h=-t^2,k=t\Rightarrow k^2=-h \text{ or } y^2=-x$$

$$k^2=-h\Rightarrow \frac{\alpha}{\sin^2\theta}=\frac{-4\sqrt{\alpha}}{\cos\theta}\ \Rightarrow\ \sqrt{\alpha}=-\frac{4\sin^2\theta}{\cos\theta}=6$$

Any normal to parabola is

$y = mx - 4m - 2m^3$

$\Rightarrow\ \ 2m^3 + (4 - h)m = 0 \Rightarrow h = 6$

**37. (A) → (3), (B) → (2), (C) → (1), (D) → (4)**

**(A)** $xyz = 3^5$ number of solution $(x, y, z)$ is
${}^{3+5-1}C_5 = 21$

**(B)** Number of terms $= {}^{6+3-1}C_{3-1} = 28$

**(C)** $x^2 + x - 400 \leq 0\ \Rightarrow\ x(x + 1) \leq 400$

$\therefore\ \ $ The numbers which satisfied the inequation are 1, 2, 3, ..., 19. Thus, number of solutions = 19

**(D)** Given, $x + y + z = 10$, where $x, y, z \in N$.

Number of solutions = coefficient of $t^{10}$ in $(t + t^2 + ... + t^{10})^3$

$$= \text{coefficient of } t^{10} \text{ in } t^3\left(\frac{1-t^{10}}{1-t}\right)^3$$

$= $ coefficient of $t^7$ in $(1 + {}^3C_1t + {}^4C_2t^2 + ..) = {}^9C_7 = 36$

**38. (A) → (3), (B) → (1), (C) → (2), (D) → (4)**

**(A)** $x^2 - ax + a > 0\ \forall\ x \in R,\ D < 0 \Rightarrow a^2 - 4a < 0$, $a \in (0, 4)$.

**(B)** Let $f(x)=x^3-3x+\dfrac{a}{2},\ f'(x)=3x^2-3=0\Rightarrow x=\pm1$

$\Rightarrow\ \ f(1)f(-1) < 0.$

## WBJEE 2021 likely in July, delay can affect admission process

The West Bengal Joint Entrance Examination, WBJEE 2021 is likely to be conducted on July 11. The delay is due to the Covid-19 pandemic situations and state assembly elections, said chairman Malayendu Saha. "As schools, colleges are yet to open, it will be difficult for us to get the exam centres. Also, there is assembly elections, therefore we decided to conduct WBJEE in July," he said.

Last year, the state JEE was held on February 2, following government direction as most of the colleges, government and private, were running with vacant seats for years. According to academicians and experts, this year, the West Bengal Joint Entrance Examination Board's decision to conduct exam late in July, months after JEE Main can impact the admission process.

For latest update visit *www.wbjeeb.nic.in.*

$$\Rightarrow \left(1-3+\frac{a}{2}\right)\left(-1+3+\frac{a}{2}\right)<0 \Rightarrow \left(\frac{a}{2}-2\right)\left(\frac{a}{2}+2\right)<0$$

$$\Rightarrow -2<\frac{a}{2}<2 \Rightarrow a\in(-4, 4) \Rightarrow |a|\in[0, 4)$$

**(C)** Let $f(x) = x^3 + ax^2 + x + 1, f'(x) = 3x^2 + 2ax + 1,$ $f'(x) \geq 0 \Rightarrow 4a^2 - 12 \leq 0 \Rightarrow a^2 \leq 3$

**(D)** $x^2 + 3ax + a^2 - 9 = 0$

Product of roots $= a^2 - 9 < 0 \Rightarrow a^2 < 9 \Rightarrow a \in (-3, 3)$

**39. (A) → (3), (B) → (2), (3), (C) → (1)**

**(A)** $\frac{dy}{dx}=2x+b\geq 0\,\forall x\in[1,3]\Rightarrow 2+b\geq 0\Rightarrow b\in[-2,\infty)$

**(B)** India played 4 matches, where maximum point in each match can be earned is 2. So, maximum point in 4 matches is 8. Getting atleast 7 points means 7 or 8.

$\therefore$ Required prob. $= P(7) + P(8)$;

$P(7) = {}^4C_1(0.35) \times (0.5)^3 = 0.175; P(8) = (0.5)^4 = 0.0625$

$\therefore$ Required prob. $= 0.2375$

**(C)** The function is defined for all values of $x$ for which $|x| - x > 0 \Rightarrow |x| > x$. If $x \geq 0, |x| = x$

$\therefore$ All non-negative values of $x$ shall not be in domain of $x$. But if $x$ is negative, then $|x| = -x > x$

Thus, $x < 0, x < 0$ and so domain is $(-\infty, 0)$.

**40. (1) :** In given $\Delta ABC$ both $\frac{A}{2}$ and $\frac{B}{2}$ lie strictly between $\left(0,\frac{\pi}{2}\right)$ and $\sin x$ is always increasing in $\left(0,\frac{\pi}{2}\right)$ whereas $\cos x$ is always decreasing throughout $\left(0,\frac{\pi}{2}\right)$.

So if $\frac{A}{2}>\frac{B}{2}\Rightarrow \sin\frac{A}{2}>\sin\frac{B}{2}\Rightarrow x_1>x_2$

and $\frac{1}{x_3}>\frac{1}{x_4}$. Now, $x_1^{2007}x_4^{2006}=x_2^{2007}x_3^{2006}$ is not valid.

Similarly for $\frac{A}{2}<\frac{B}{2}$

$\sin\frac{A}{2}<\sin\frac{B}{2}\Rightarrow x_1<x_2$ and $\frac{1}{x_3}<\frac{1}{x_4}$. Again equality is not possible.

Therefore only possible case is when $\frac{A}{2}=\frac{B}{2}$

$\Rightarrow x_1=x_2$ and $\frac{1}{x_3}=\frac{1}{x_4}$.

Hence in this case $\Delta ABC$ is isosceles with $\angle ABC = \angle CAB$

**41. (1) :** $I_n=\left(\frac{e^{-x}(\sin x)^n}{-1}\right)_0^\infty+\int_0^\infty n(\sin x)^{n-1}\cos x e^{-x}dx$

$$=0+n\left(\frac{(\sin x)^{n-1}\cos x e^{-x}}{-1}\right)_0^\infty$$

$$+n\int_0^\infty\left[(\sin x)^{n-1}(-\sin x)+(\cos x)(n-1)\cos x(\sin x)^{n-2}\right]e^{-x}dx$$

$$=n\int_0^\infty\left[-(\sin x)^n+(n-1)(1-\sin^2 x)(\sin x)^{n-2}\right]e^{-x}dx$$

$$=\frac{n(n-1)}{n^2+1}I_{n-2},\ \frac{101}{90}\frac{I_{10}}{I_8}=\frac{101}{90}\times\frac{10\times 9}{10^2+1}=1$$

**42. (2) :** A.M. $=\frac{100+n}{2}$, G.M. $=\sqrt{100\times n}$,

H.M. $=\frac{200n}{100+n}$

Let $n = 4k^2$ as for A.M and G.M we need a even perfect square.

$\therefore$ H.M. $=\frac{800k^2}{100+4k^2}=\frac{200k^2}{25+k^2}$

Now, $100<\frac{200k^2}{25+k^2}\leq 500 \Rightarrow 2500 + 100k^2 < 200k^2$

$\Rightarrow 2500 < 100k^2$

and $200k^2 \leq 12500 + 500k^2 \Rightarrow -300k^2 \leq 12500$

$\Rightarrow k^2 = 36, 49, 64, 81, 100$, H.M. is an integer for $k^2 = 100$ only

$\Rightarrow n = 4k^2 = 4 \times 100 = 400$

◆◆

## PUZZLE CORNER

ANSWER · FEBRUARY 2021

| | | | | | |
|---|---|---|---|---|---|
| 11+ 5 | 2÷ 6 | 3 | 20× 4 | 6× 1 | 2 |
| 6 | 3− 1 | 4 | 5 | 3÷ 2 | 3 |
| 240× 4 | 5 | 6× 2 | 3 | 6 | 1 |
| 3 | 4 | 6× 1 | 7+ 2 | 30× 5 | 6 |
| 6× 2 | 3 | 6 | 1 | 4 | 9+ 5 |
| 8+ 1 | 2 | 5 | 2÷ 6 | 3 | 4 |

**Exam Dates 2021**
15th to 18th March
27th to 30th April; 24th to 28th May

# Challenging PROBLEMS ON JEE Main

**1.** A random variable $X$ takes values 0, 1, 2, 3, ….. and $P(X = r)$ is proportional to $(r+1)\left(\frac{1}{5}\right)^r$ then which of the following is not true?

(a) $P(X=0)=\frac{16}{25}$ (b) $P(X\geq 1)=\frac{9}{25}$

(c) $P(X\leq 1)=\frac{112}{125}$ (d) $E(X)=\frac{32}{45}$

**2.** Let '$a$' be a factor of 120, then the number of positive integral solutions of $x_1x_2x_3 = a$ is

(a) 160 (b) 320 (c) 480 (d) 960

**3.** Let $z$ be a complex number such that $\left|3z+\frac{1}{z}\right|=1$ and $\arg(z) = \phi$ then the maximum value of $\cos^2\phi$ equals

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$

**4.** If a complex number $\alpha$ satisfies the equation $(39z - 12\,\bar{z}\,)^4 = -1$ then the value of

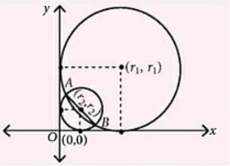
$\left[\frac{|\text{Re}(\alpha)|^{-1}+|\text{Im}(\alpha)|^{-1}}{13}\right]$ (where [ · ] is G.I.F.) is

(a) 5 (b) 8 (c) 10 (d) 12

**5.** A circle with centre $Q$ having double contact with parabola $y^2=(2+\sqrt{3})x$ passes through its focus $S$. If tangent at their point of contact $P$ intersect its axis at $R$, then radius of inscribed circle of triangle $PSR$ is

(a) $2\sqrt{3}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

**6.** Let two circles of radii $r_1$ and $r_2(r_1 > r_2)$ in the first quadrant are tangent to co-ordinate axes. If the length of common chord of circles is maximum, then find the value of $\frac{r_1}{r_2}$ is



(a) 3 (b) 5 (c) 4 (d) 2

**7.** In the $xy$-plane three distinct lines $l_1$, $l_2$, $l_3$ are concurrent at $M(\lambda, 0)$. Also the lines $l_1, l_2, l_3$ are normals to the parabola $y^2 = 6x$ at the points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ respectively. Then

(a) $\lambda < -5$ (b) $\lambda > 3$

(c) $-5 < \lambda < -3$ (d) $0 < \lambda < 3$

**8.** Consider a parabola $y=\frac{x^2}{4}$ and the point $F(0,1)$. Let $A_1(x_1, y_1)$, $A_2(x_2, y_2)$, $A_3(x_3, y_3)$, ……., $A_n\ (x_n, y_n)$ are '$n$' points on the parabola such that $x_k > 0$ and $\angle OFA_k\ =\frac{k\pi}{2n}(k = 1, 2, 3,….., n)$. Then the value of $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}FA_k$, is

(a) $\frac{2}{\pi}$ (b) $\frac{4}{\pi}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

**9.** Tangents drawn from the point $P(2, 3)$ to the circle $x^2 + y^2 - 8x + 6y + 1 = 0$ touch the circle at the points $A$ and $B$. The circumcircle of the triangle $PAB$ cuts the director circle of ellipse $\frac{(x+5)^2}{9}+\frac{(y-3)^2}{b^2}=1$ orthogonally. Then the value of $b^2$ is

(a) 50 (b) 54 (c) 60 (d) 64

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**10.** If the two points (0, 0) and (1, 1) lie on the same side of the line $a^2x + 2b(a + b + c)y + 4 = 0 \ \forall\ a, b \in R$ then the maximum value of $c$, is

(a) 1 (b) 2 (c) 3 (d) 4

**11.** Number of solutions satisfying the equation, $3x^2 - 2x^3 = \log_2(x^2 + 1) - \log_2 x$ is

(a) 1 (b) 2 (c) 3 (d) infinite

**12.** Consider the function $y = f(x)$ satisfying $y = \cos(x - y)$, $x \geq 0$ and if $P$ and $Q$ are two nearest stationary points from origin, then $\overline{PQ}$ equals

(a) $\sqrt{\pi^2+4}$ (b) $\sqrt{\pi^2-4\pi+8}$

(c) $\sqrt{\pi^2+4\pi+8}$ (d) $\sqrt{\pi^2-4}$

**13.** If $x \in R$ the maximum value of $\sqrt{x^4-3x^2-6x+13}-\sqrt{x^4-x^2+1}$ equals

(a) $\sqrt{7}$ (b) $\sqrt{10}$ (c) 3 (d) 4

**14.** If $x$, $y$, $z$ are positive real numbers then the minimum value of $\frac{x^4+y^4+z^2}{xyz}$ is $k_1$ and minimum value of $\frac{\sec^4\alpha}{\tan^2\beta}+\frac{\sec^4\beta}{\tan^2\alpha}, \alpha, \beta \in \left(0, \frac{\pi}{2}\right)$ is $k_2$. The sum of digits of $k_1^2 + k_2^2$ equals

(a) 8 (b) 12 (c) 10 (d) 9

**15.** Let $f: R \to R^+$ be a differentiable function satisfying $f'(x) = 2f(x)\ \forall\ x \in R$. Also, $f(0) = 1$ and $g(x) = f(x)\cdot\cos^2 x$. If $n_1$ represent number of points of local maxima of $g(x)$ in $[-\pi, \pi]$ and $n_2$ is the number of points of local minima of $g(x)$ in $[-\pi, \pi]$ and $n_3$ is the number of points in $[-\pi, \pi]$, where $g(x)$ attains its global minimum value, then the value of $(n_1 + n_2 + n_3)$ is

(a) 7 (b) 8 (c) 5 (d) 4

**16.** Let $f(x)=\frac{x^4-\lambda x^3-3x^2+3\lambda x}{x-\lambda}$. If range of $f(x)$ is the set of entire real numbers, then the true set in which $\lambda$ lies is

(a) $[-2, 2]$ (b) $[0, 4]$

(c) $(1, 3)$ (d) None of them

**17.** Let $f: R \to R$ be a differentiable function which satisfies $f(x)=\int_0^x |x-f(x)|\,dx \forall x \in R$, then which of the following is not correct?

(a) $f(x)$ is a monotonic function

(b) $f(x)$ is an onto function

(c) $f(x)$ is an odd function

(d) $y = x$ is tangent to $f(x)$

**18.** Let $f(x)$ be a function such that $f(x) = x - [x]$, where $[x]$ is the greatest integer less than or equal to $x$. Then the number of solutions of the equation $f(x)+f\left(\frac{1}{x}\right)=1$ is (are)

(a) 0 (b) 1 (c) 2 (d) infinite

**19.** A monic quadratic trinomial $P(x)$ is such that $P(x) = 0$ and $P(P(P(x))) = 0$ have a common root, then

(a) $P(0)\cdot P(1) > 0$ (b) $P(0)\cdot P(1) < 0$

(c) $P(0)\cdot P(1) = 0$ (d) None of these

**20.** If the sum of the maximum value and the minimum value of $y = \log_2(x^4 + x^2 + 1) - \log_2(x^4 + x^3 + 2x^2 + x + 1)$ can be expressed in form $((\log_2 m) - n)$, where $m$ and $n$ are relatively prime positive integers, then the value of $(m + n)$ is

(a) 5 (b) 2 (c) 3 (d) 4

## Numerical Value Type

**21.** Let $f: R \to R$ be defined as $f(x) = x + \cos x + 2$ and $g(x)$ be the inverse function of $f(x)$. Then value of $(g'(3) + g''(3))$ is ____________.

**22.** The absolute value of the sum of first 20 terms of series, if $S_n=\frac{n+1}{2}$ and $\frac{T_{n-1}}{T_n}=\frac{1}{n^2}-1$ where $n$ is odd, given $S_n$ and $T_n$ denotes sum of first n terms and $n^{\text{th}}$ term of the series, is ____________.

**23.** The number of ordered pair(s) $(a, b)$ for which the function $f(x) = \text{sgn}((x^2 - ax + 1)(bx^2 - 2bx + 1))$ is discontinuous at exactly one point(where $a$, $b$ are integer and $\text{sgn}(x)$ denotes signum function of $x$) is/are ____________.

**24.** If $a_1$, $a_2$, $a_3$ are the last three digits of $17^{256}$ respectively, then the value of $4a_1 - 2a_2 - a_3$ is equal to____________.

**25.** Consider the cubic polynomial $p(x) = x^3 - ax^2 + bx + c$. If the equation $p(x) = 0$ has three distinct integral roots such that $p(6) = 3$, then least possible value of '$a$' is ____________.

**26.** Let $P(x) = x^2 + bx + c$, where $b$ and $c$ are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then the value of $P(1)$ is ____________.

**27.** If $x^3 - 3x^2 - x + 4 = 0$ and $ax^3 - bx^2 - cx + 5 = 0$ have one root common, where $a, b, c \in Q$, then the value of $-\frac{b^2}{ac}$ is____________.

**28.** If $x, y, z$ are positive real numbers such that $y, z, x$ are in G.P. and $x + y + z = xyz$, then the maximum value of $\frac{76}{x^4+x^2+7}$ is ____________.

**29.** In a $\Delta ABC$, $x+y+2=0$ is the perpendicular bisector of side $AB$ and it meets $AB$ at $(-1, -1)$. If $x - y - 1 = 0$ is $\perp r$ bisector of side $AC$ and it meets $AC$ at $(2, 1)$, $P$ is mid point of $BC$ and distance of $P$ from orthocentre of $\Delta ABC$ is $k$. Then $k^2 - 6$ is ____________ .

**30.** Let $a_1, a_2, a_3, a_4$ and $b$ are positive real number such that $b + \sum_{k=1}^{4} a_k = 8$ and $b_2 + \sum_{k=1}^{4} a_k^2 = 16$. The number of possible integral values $b$ can take is ____________ .

## SOLUTIONS

**1. (d) :** $P(X=r) \propto (r+1)\left(\frac{1}{5}\right)^r$

$$\Rightarrow P(X=r)=k(r+1)\left(\frac{1}{5}\right)^r$$

$$\Rightarrow \sum_{r=0}^{\infty} P(X=r)=1 \Rightarrow \sum_{r=0}^{\infty} k(r+1)\left(\frac{1}{5}\right)^r=1$$

$$\Rightarrow k\left[1+2\left(\frac{1}{5}\right)+3\left(\frac{1}{5}\right)^2+\ldots\right]=1 \Rightarrow k\left(1-\frac{1}{5}\right)^{-2}=1$$

$$\Rightarrow k=\frac{16}{25}$$

$$\therefore\ E(X)=\sum_{r=0}^{\infty} rP(X=r)=\sum_{r=0}^{\infty} kr(r+1)\left(\frac{1}{5}\right)^r$$

$$E(X)=k\left[0+1\cdot 2\left(\frac{1}{5}\right)+2\cdot 3\left(\frac{1}{5}\right)^2+3\cdot 4\left(\frac{1}{5}\right)^3+\ldots\ldots\right]$$

$$\frac{1}{5}E(X)=k\left[\qquad\qquad 1\cdot 2\left(\frac{1}{5}\right)^2+2\cdot 3\left(\frac{1}{5}\right)^3+\ldots\right]$$

$$\Rightarrow \frac{4}{5}E(X)=k\left[2\left(\frac{1}{5}\right)+4\left(\frac{1}{5}\right)^2+6\left(\frac{1}{5}\right)^3+\ldots\ldots\right]$$

$$\Rightarrow E(X)=\frac{5}{4}\times\frac{16}{25}\times 2\times\frac{1}{5}\times\frac{25}{16}=\frac{1}{2}$$

$$P(X\ge 1)=1-P(X=0)=1-k=\frac{9}{25}$$

$$P(X\le 1)=P(X=0)+P(X=1)=k\left[1+\frac{2}{5}\right]=\frac{16}{25}\times\frac{7}{5}=\frac{112}{125}$$

**2. (b) :** $x_1x_2x_3x_4 = 120$, where $120/a = x_4$
$= 2^3 \cdot 3 \cdot 5$

For $2^3$, the no. of ways $= {}^6C_3 = 20$
For 3, the no. of ways $= {}^4C_3 = 4$
Similarly, for 5, the no. of ways $= {}^4C_3 = 4$
Total number of integral solutions $= 20 \times 4 \times 4 = 320$

**3. (d) :** $z = r(\cos\phi + i\sin\phi)$

$$\Rightarrow \left|\left(3r+\frac{1}{r}\right)\cos\phi+i\left(3r-\frac{1}{r}\right)\sin\phi\right|=1$$

$$\Rightarrow \cos^2\phi\left\{\left(3r+\frac{1}{r}\right)^2-\left(3r-\frac{1}{r}\right)^2\right\}=1-\left(3r-\frac{1}{r}\right)^2$$

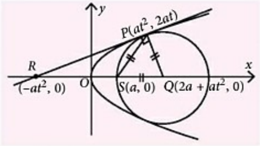
$$\Rightarrow 12\cos^2\phi\le 1 \Rightarrow \cos^2\phi\le\frac{1}{12}$$

**4. (b) :** Let $\alpha=a+ib \Rightarrow 27a+(51b)i=e^{i\frac{(2k+1)\pi}{4}}$

Solutions : $a=\pm\frac{1}{27\sqrt{2}},\ b=\pm\frac{1}{51\sqrt{2}}$

Given expression $=\left[\frac{27\sqrt{2}+51\sqrt{2}}{13}\right]=8$

**5. (d) :**



Since $Q$ is centre of circle
$\therefore\ SQ = SP = PQ$ (radius) $\Rightarrow \angle PQS = 60°$
Now, In $\Delta PSR$, $\angle R = 30°$, $PS = SR$

$$\Rightarrow \tan 30°=\frac{\sqrt{t^2+1}}{2t} \Rightarrow t=\pm\sqrt{3}$$

Thus, we have $P(3a, 2\sqrt{3}a),\ S(a,0),\ R(-3a,0)$

$$\text{Now, } r=\frac{\Delta}{s}=\frac{\frac{1}{2}\cdot RS\cdot PS\cdot\frac{\sqrt{3}}{2}}{\frac{1}{2}(RS+PS+RP)}=\frac{\sqrt{3}}{2}$$

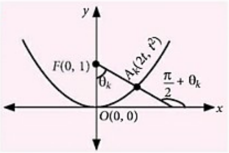
**6. (a) :** Let equation of circles be
$S \equiv (x - r_1)^2 + (y - r_1)^2 = r_1^2$ ...(i)
and $S' \equiv (x - r_2)^2 + (y - r_2)^2 = r_2^2$ ...(ii)
where $r_1 > r_2$
Equation of common chord is $S - S' = 0$ given by
$2(x + y) = r_1 + r_2$ ...(iii)
For maximum length of common chord, above equation (iii) must pass through the centre $(r_2, r_2)$ of the smaller circle.

$$\Rightarrow 4r_2=r_1+r_2 \Rightarrow 3r_2=r_1 \Rightarrow \frac{r_1}{r_2}=3$$

**7. (b) :** Any normal is $y = mx - 2am - am^3$,
where $a=\frac{3}{2}$
$\therefore\ (\lambda, 0) \Rightarrow 0 = m\lambda - 2am - am^3$

$$\Rightarrow m=0, m^2=\frac{\lambda}{a}-2>0 \Rightarrow \lambda>2a \Rightarrow \lambda>3$$

**8. (b) :**



Let $A_k = (2t, t^2)$

$\therefore$ Slope of $FA_k = \dfrac{t^2-1}{2t-0} = \tan\left(\dfrac{\pi}{2}+\theta_k\right)$

$\therefore$ $\tan\theta_k = \dfrac{2t}{1-t^2} = \tan(2\phi)$ (say)

$\Rightarrow$ $\phi = \dfrac{\theta_k}{2} = \dfrac{k\pi}{4n}$, where $\tan\phi = t$

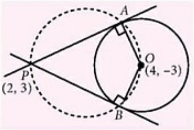
Also, $FA_k = \sqrt{(t^2-1)^2+(2t)^2}$

$= t^2 + 1 = 1 + \tan^2\phi = \sec^2\left(\dfrac{k\pi}{4n}\right)$

$$\therefore \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} FA_k = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sec^2\left(\frac{\pi}{4}\left(\frac{k}{n}\right)\right)$$

$$= \int_0^1 \sec^2\left(\frac{\pi x}{4}\right)dx = \frac{4}{\pi}\left[\tan\frac{\pi x}{4}\right]_0^1 = \frac{4}{\pi}$$

**9. (b) :** Center of the given circle is $O(4, -3)$.



The circumcircle of triangle *PAB* will circumscribe the quadrilateral *PBOA* also, hence one of the diameters must be *OP*.

Therefore, equation of circumcircle of triangle *PAB* will be $(x-2)(x-4)+(y-3)(y+3)=0$

$\Rightarrow$ $x^2+y^2-6x-1=0$ (i)

Director circle of given ellipse will be

$(x+5)^2+(y-3)^2 = 9+b^2$

$\Rightarrow$ $x^2+y^2+10x-6y+25-b^2=0$ (ii)

From (i) and (ii), by applying the condition of orthogonal intersection, we get

$2(-3\cdot 5+0\cdot(-3)) = -1+25-b^2$

$\Rightarrow$ $-30 = 24-b^2$

Hence, $b^2 = 54$

**10. (b) :** Point may lie on the same side of the line or atleast one on the line, so

$4[a^2+2b(a+b+c)+4] \geq 0$

$\Rightarrow$ $a^2+(2b)a+2b(b+c)+4 \geq 0$

$\because$ $D \leq 0$, $4b^2-4[2b^2+2bc+4] \leq 0$

$\Rightarrow$ $-b^2-2bc-4 \leq 0$

$\Rightarrow$ $b^2+2bc+4 \geq 0$

$\because$ $D \leq 0$; $4c^2-16 \leq 0$, $c^2-4 \leq 0$, $-2 \leq c \leq 2$

$\therefore$ Max. value of $c = 2$

**11. (a) :** Compare the greatest value of the function appearing on the left hand side of the equation with the least value appearing on the right hand side.

Note that L.H.S has a maximum value 1 at $x = 1$ and R.H.S side has a minimum value at $x = 1$. So, $x = 1$ can be the only solution.

**12. (b) :** $P(1,1), Q(\pi-1,-1) \Rightarrow PQ = \sqrt{\pi^2-4\pi+8}$

**13. (b) :** Given expression $= \sqrt{(x^2-2)^2+(x-3)^2}$

$-\sqrt{(x^2-1)^2+(x-0)^2}$

Let $P(x, x^2), A(3, 2)B(0, 1)$, then $|PA|-|PB| \leq |AB| = \sqrt{10}$

**14. (d) :** Using A.M. ≥ G.M. for $x^4, y^4, \dfrac{z^2}{2}, \dfrac{z^2}{2}$, we get $k_1 = 2\sqrt{2}$.

Using, $a^2+b^2 \geq 2ab$, we get

Given expression $\geq \dfrac{2\sec^2\alpha\sec^2\beta}{\tan\beta\tan\alpha} \geq \dfrac{8}{\sin 2\alpha\sin 2\beta} \geq 8$

$\Rightarrow$ $k_1^2+k_2^2 = 8+64 = 72$

$\therefore$ Sum of digits = 9

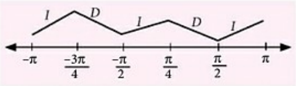
**15. (b) :** Given $f'(x) = 2f(x)$ $\therefore$ $\dfrac{f'}{f} = 2 \Rightarrow f(x) = Ae^{2x}$

$f(0) = 1 = A$

$\therefore$ $f(x) = e^{2x}$

Now, $g(x) = e^{2x}\cdot\cos^2 x$

$g'(x) = e^{2x}(-2\cos x\sin x + 2\cos^2 x)$



$g'(x) = 2\cos x\, e^{2x}(\cos x - \sin x)$

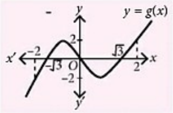
$g'(x) = 0 \Rightarrow x = \dfrac{-3\pi}{4}, \dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{-\pi}{2}$

$\therefore$ Points of maxima are $\dfrac{-3\pi}{4}, \dfrac{\pi}{4}$ and $\pi$, points of minima are $-\pi, \dfrac{-\pi}{2}, \dfrac{\pi}{2}$ and global minimum value occurs at $\pm\dfrac{\pi}{2}$, which is zero.

Hence, $n_1 = 3, n_2 = 3, n_3 = 2, \Rightarrow n_1+n_2+n_3 = 8$

**16. (a) :** Here $f(x)=\dfrac{(x^3-3x)(x-\lambda)}{x-\lambda}$

Consider $g(x) = x^3 - 3x$



From graph it is clear that $\lambda \in [-2, 2]$.

**17. (d) :** $f'(x) = |x - f(x)|$

$\Rightarrow\ f'(x) + f(x) = x$ if $x \geq f(x)$

and $f'(x) - f(x) = -x$ if $x \leq f(x)$

$\Rightarrow\ f(x) = (x - 1) + e^{-x}$ if $x \geq f(x)$

and $f(x) = (x + 1) - e^{x}$ if $x \leq f(x)$

$$\Rightarrow\ f(x)=\begin{cases} x-1+e^{-x} & x\geq 0 \\ x+1-e^{x} & x\leq 0 \end{cases}$$

**18. (d) :** Given, $f(x) = x - [x], x \in R - \{0\}$

Now $f(x)+f\left(\dfrac{1}{x}\right)=1 \ \therefore\ x-[x]+\dfrac{1}{x}-\left[\dfrac{1}{x}\right]=1$

$$\Rightarrow\ \left(x+\frac{1}{x}\right)-\left([x]+\left[\frac{1}{x}\right]\right)=1$$

$$\Rightarrow\ \left(x+\frac{1}{x}\right)=[x]+\left[\frac{1}{x}\right]+1 \qquad \ldots\text{(i)}$$

Clearly ,R.H.S is an integer

$\therefore$ L.H.S. is also an integer

Let $x+\dfrac{1}{x}=k$, an integer $\Rightarrow x^2 - kx + 1 = 0$

$$\therefore\ x=\frac{k\pm\sqrt{k^2-4}}{2}$$

For real values of $x$, $k^2 - 4 \geq 0 \Rightarrow k \geq 2$ or $k \leq -2$

We also observe that $k = 2$ and $-2$ does not satisfy equation (i).

$\therefore$ The equation (i) will have solutions if $k > 2$ or $k < -2$, where $k \in Z$.

Hence equation (i) has infinite number of solutions.

**19. (c) :** Let $P(x) = x^2 + ax + b$, and let $r$ be a common root.

$\therefore\ P(r) = 0$ and $P(P(P(r))) = 0$ *i.e.*, $P(P(0)) = 0, P(0) = b$

$\therefore\ P(b) = 0 \Rightarrow b^2 + ab + b = 0$

$\Rightarrow\ b(a + b + 1) = 0 \Rightarrow P(0)\cdot P(1) = 0.$

**20. (a) :**

$$y=\log_2\left(\frac{(x^2+x+1)(x^2-x+1)}{x^4+x^3+2x^2+x+1}\right)=\log_2\left(\frac{x^2-x+1}{x^2+1}\right)$$

Assume $z=\dfrac{x^2-x+1}{x^2+1}$

$\Rightarrow\ (z - 1)x^2 + x + (z - 1) = 0.$

As $x \in R$, so $D\geq 0 \Rightarrow 1-4(z-1)^2\geq 0$

$$\Rightarrow\ (z-1)^2\leq\frac{1}{4}\Rightarrow\ \frac{1}{2}\leq z\leq\frac{3}{2}$$

So, $y_{\min}=\log_2\left(\dfrac{1}{2}\right)=-1$

and $y_{\max}=\log_2\left(\dfrac{3}{2}\right)=(\log_2 3-1)$

Hence $y_{\min} + y_{\max} = \log_2 3 - 2 \equiv ((\log_2 m) - n)$

$\therefore\ m = 3, n = 2$. Hence $(m + n) = 3 + 2 = 5$

**21. (2) :** $f(x) = x + \cos x + 2$

$f'(x) = 1 - \sin x \Rightarrow f'(0) = 1$

Now, $\dfrac{dx}{dy}=\dfrac{1}{\dfrac{dy}{dx}}\Rightarrow g'(y)=\dfrac{1}{f'(x)}$

When $y = 3$ then $x = 0$

$$\therefore\ g'(3)=\frac{1}{f'(0)}=1$$

$$\text{Again } \frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right)=\frac{d}{dx}\left(\frac{1}{\frac{dy}{dx}}\right)\cdot\frac{dx}{dy}=\frac{-1}{\left(\frac{dy}{dx}\right)^3}\cdot\frac{d^2y}{dx^2}$$

$$\Rightarrow\ g''(y)=\frac{-f''(x)}{(f'(x))^2}\Rightarrow\ g''(3)=\frac{-f''(0)}{(f'(0))^3}=\frac{1}{1}$$

$\Rightarrow\ g''(3) = 1$

Hence, $g'(3) + g''(3) = 1 + 1 = 2$

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**22. (430) :** We have $S_n - S_{n-2} = T_n + T_{n-1}$

(Taking $n$ to be odd)

$$\Rightarrow T_{n-1}\left(1+\frac{T_n}{T_{n-1}}\right)=\left(\frac{n+1}{2}\right)-\left(\frac{n-1}{2}\right)=1$$

$$\left(\text{as } S_n=\frac{n+1}{2}, \text{ if } n \text{ is odd}\right)$$

$$\Rightarrow T_{n-1}\left(1+\frac{n^2}{1-n^2}\right)=1 \Rightarrow T_{n-1}=-(n^2-1), \text{ when } n \text{ is odd.}$$

Also, $S_m = S_{m-1} + T_m$

If $m$ is even then $(m \pm 1)$ is odd.

$$\therefore\ S_m=\frac{m}{2}-((m+1)^2-1)=\frac{-m(2m+3)}{2}$$

$$\Rightarrow\ |S_{20}|=\left|\frac{-20}{2}(43)\right|=430.$$

**23. (6) :** We have, $f(x) = \text{sgn}[(x^2 - ax + 1)(bx^2 - 2bx + 1)]$

For exactly one point of discontinuity,

$(x^2 - ax + 1)(bx^2 - 2bx + 1) = 0$ at exactly one value of $x$.

Let $P(x) = x^2 - ax + 1$ ...(1)

and $Q(x) = bx^2 - 2bx + 1$ ...(2)

and $D_1$ and $D_2$ are the discriminant of equation (1) and (2) respectively.

**Case I :** $D_1 = 0$ and $D_2 < 0$

$a^2 - 4 = 0$ and $4b^2 - 4b < 0$

$a = \pm 2$ and $b \in (0, 1)$ rejected because there is no integral value of $b$.

**Case II :** $D_1 < 0$ and $D_2 = 0$

$a \in (-2, 2)$ and $b = 0, 1$

$\Rightarrow\ a = -1, 0, 1$ and $b = 0$ is rejected.

$\therefore$ Number of ordered pairs are 3.

**Case III :** $D_1 = 0$ and $D_2 = 0$

$a = -2, 2$ and $b = 0, 1$

For $b = 0$, $a = -2$ and 2 and for $b = 1$, $a = 2$

$\therefore$ Number of ordered pairs are 3.

$\therefore$ Total number of ordered pairs are 6.

**24. (7) :** $17^{256} = (290 - 1)^{128} = 1000k + {}^{128}C_{126}(290)^2 - {}^{128}C_{127}(290) + 1$

$\therefore\ a_1 = 6, a_2 = 8, a_3 = 1.$

**25. (13) :** Let $x^3 - ax^2 + bx + c = (x - \alpha)(x - \beta)(x - \gamma)$

$p(6) = 3, \alpha, \beta, \gamma \in Z \Rightarrow 3 = (6 - \alpha)(6 - \beta)(6 - \gamma)$

$\Rightarrow\ 6 - \alpha = 3, 6 - \beta = 1, 6 - \gamma = 1$

or $6 - \alpha = -3,\ 6 - \beta = -1, 6 - \gamma = 1$

or $6 - \alpha = -1, 6 - \beta = -1, 6 - \gamma = 3$

**26. (4) :** Since $P(x)$ divides both of them, therefore $P(x)$ will also divide $(3x^4 + 4x^2 + 28x + 5) -3(x^4 + 6x^2 + 25) = -14x^2 + 28x - 70 = -14(x^2 - 2x + 5)$, which is a quadratic. Hence, $P(x) = x^2 - 2x + 5$

$\therefore\ P(1) = 4$

**27. (9) :** $x^3 - 3x^2 - x + 4 = 0$ has all 3 roots irrational and $a, b, c \in Q$ if one roots is common, then

$$\frac{a}{1}=\frac{b}{-3}=\frac{c}{-1}=\frac{5}{4}$$

**28. (4) :** Let $z = yr$ and $x = yr^2 \Rightarrow yr^2 + y + yr = (yr^2)(y)(yr)$

$$\Rightarrow\ y^2=\frac{1+r+r^2}{r^3}$$

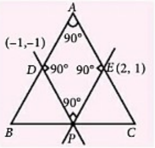
Now $x^4 + x^2 + 7 = y^4r^4 + y^2r^2 + 7$

$$=r^4\frac{(1+r+r^2)^2}{r^6}+\frac{r^2(1+r+r^2)}{r^3}+7$$

$$=\left(1+r+\frac{1}{r}\right)^2+\left(1+r+\frac{1}{r}\right)+7\geq 19$$

$\therefore$ Given, expression $\leq \frac{76}{19} = 4 \Rightarrow$ Max. value = 4

**29. (7) :** Clearly $x + y + 2 = 0$ and $x - y - 1 = 0$ are perpendicular to each other



$\therefore\ \angle BAC = 90°$

$\therefore$ $A$ is the orthocentre of $\Delta ABC$.

Mid point of $BC = P$

$$= \text{Circumcentre} = \left(\frac{-1}{2},\frac{-3}{2}\right)$$

$\therefore\ PA = DE = \sqrt{9+4} = \sqrt{13}$

**30. (3) :** Given $b + (a_1 + a_2 + a_3 + a_4) = 8$

or $a_1 + a_2 + a_3 + a_4 + a_4 = 8 - b$ ...(i)

Also, $b^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 = 16$

or $a_1^2 + a_2^2 + a_3^2 + a_4^2 = 16 - b^2$ ...(ii)

Since, $a_1, a_2, a_3, a_4 \in R^+$.

$$\therefore\ \sqrt{\frac{a_1^2+a_2^2+a_3^2+a_4^2}{4}}\geq\frac{a_1+a_2+a_3+a_4}{4}$$

$$\Rightarrow\ 16-b^2\geq\frac{(8-b)^2}{4}\Rightarrow 64-4b^2\geq 64+b^2-16b$$

$\Rightarrow\ 5b^2 - 16b \leq 0 \Rightarrow b(5b - 16) \leq 0.$

Thus $0 \leq b \leq \frac{16}{5}$.

Hence, integral values of $b = 1, 2, 3$

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# OLYMPIAD CORNER

1. Evaluate the product $\prod_{k=0}^{2^{1999}}\left(4\sin^2\frac{k\pi}{2^{2000}}-3\right)$.

2. Given a triangle $ABC$, show how to construct, with straight, edge and compass, a triangle $A'B'C'$ with minimal area such that $A'$, $B'$, $C'$ lie on $AB$, $BC$, $CA$, respectively, $\angle B'A'C' = \angle BAC$, and $\angle A'C'B' = \angle ACB$.

3. Let $B_1$ and $C_1$ be points on the sides $AC$ and $AB$ of triangle $ABC$. Lines $BB_1$ and $CC_1$ intersect at point $D$. Prove that a circle can be inscribed inside quadrilateral $AB_1DC_1$ if and only if the incircles of the triangles $ABD$ and $ACD$ are tangent to each other.

4. Let $x$, $y$, $z$ be non negative real numbers such that $x + y + z = 1$. Prove that $x^2y+y^2z+z^2x\le\frac{4}{27}$, and determine when equality occurs.

5. Show that there exists a triangle $ABC$ for which, with the usual labelling of sides and medians, it is true that $a \neq b$ and $a + m_a = b + m_b$. Further show that there exists a number $k$ such that for each such triangle $a+m_a=b+m_b=k(a+b)$. Finally, find all possible ratios $a : b$ of the sides of these triangles.

6. For any given positive integer $n$, determine (as a function of $n$) the number of ordered pairs $(x, y)$ of positive integers such that $x^2 - y^2 = 10^2 \cdot 30^{2n}$. Further prove that the number of such pairs is never a perfect square.

7. (a) Let $a, b \in R$, $a < b$. Prove that $a < x < b$ if and only if there exists $0 < \lambda < 1$ such that $x = \lambda a + (1 - \lambda)b$.

   (b) The function $f: R \rightarrow R$ has the property:
   $f(\lambda x + (1 - \lambda)y) < \lambda f(x) + (1 - \lambda) f(y)$
   for all $x, y \in R$, and all $0 < \lambda < 1$. Prove that one cannot find four points on the function's graph that are the vertices of a parallelogram.

## SOLUTIONS

1. For all real $x$, let $f(x)=\sin\left(\frac{x\pi}{2^{2000}}\right)$

When $k = 0$, the expression inside the parentheses equals $-3$. Using the triple angle formula $\sin(3\theta) = 4\sin^3\theta - 3\sin^3\theta$, and noting that $f(k) \neq 0$ when $1 \le k \le 2^{1999}$, we can rewrite the given product as

$$-3\prod_{k=1}^{2^{1999}}\frac{\sin\left(\frac{3k\pi}{2^{2000}}\right)}{\sin\left(\frac{k\pi}{2^{2000}}\right)} \text{ or } -3\prod_{k=1}^{2^{1999}}\frac{f(3k)}{f(k)}. \qquad \text{...(1)}$$

Observe that

$$\prod_{k=1}^{2^{1999}} f(3k)=\prod_{k=1}^{\frac{2^{1999}-2}{3}} f(3k).\prod_{k=\frac{2^{1999}+1}{3}}^{\frac{2^{2000}-1}{3}} f(3k).\prod_{\frac{2^{2000}+2}{3}}^{2^{1999}} f(3k)$$

Because $\sin\theta = \sin(\pi - \theta) = -\sin(\pi + \theta)$, we have $f(x) = f(2^{2000} - x) = -f(x - 2^{2000})$

Hence, letting $S_i = \{k, |1 \le k \le 2^{1999}, k \equiv i(\text{mod}3)\}$ for $i = 0,1,2$, the last expression equals

$$\prod_{k=1}^{\frac{2^{1999}-2}{3}} f(3k)\prod_{k=\frac{2^{1999}+1}{3}}^{\frac{2^{2000}-1}{3}} f(2^{2000}-3k).\prod_{\frac{2^{2000}+2}{3}}^{2^{1999}}(-f(3k-2^{2000}))$$

$$=\prod_{k\in S_0} f(k)\prod_{k\in S_1} f(k).\prod_{k\in S_2}(-f(k))$$

$$=(-1)\frac{2^{1999}+1}{3}\prod_{k=1}^{2^{1999}} f(k)=-\prod_{k=1}^{2^{1999}} f(k)$$

Combined with the expression in (1), this implies that the desired product is $(-3)(-1) = 3$.

2. All angles are directed modulo 180°.

For convenience, call any triangle $A'B'C'$ proper if $A'$, $B'$, $C'$ lie on lines $AB$, $BC$, $CA$, respectively, and $\Delta ABC \sim \Delta A'B'C'$ . The problem is, then, to construct the proper triangle with minimal area. Suppose we have any proper triangle, and let $P$ be the point (different from $A'$ ) where the circumcircles of triangles $AA'C'$ and $BB'A'$ meet.

Then $\angle B'PC' = 360° - \angle A'PB' - \angle C'PA'$

$= 360° - (180° - \angle CBA) - (180° - \angle BAC)$

$= 180° - \angle ACB$,

so $P$ also lies on the circumcircle of triangle $CC'B'$.

Next, $\angle PAB = \angle PC'A' = \angle B'C'A' - \angle B'C'P$

$= \angle B'CC' - \angle B'CP' = \angle PCA$,

and with similar reasoning we have

$\angle PAB = \angle PC'A' = \angle PCA = \angle PB'C' = \angle PBC$. There is a unique point $P$ (one of the Brocard points) satisfying $\angle PAB = \angle PBC = \angle PCA$, and thus $P$ is fixed – independent of the choice of triangle $A'B'C'$ . Because $P$ is the corresponding point in similar triangles $ABC$ and $A'B'C'$, we have

$$[A'B'C'] = [ABC]\left(\frac{PA'}{PA}\right)^2$$

Thus $[A'B'C']$ is minimal when $PA'$ is minimal, which occurs when $PA' \perp AB$ (and analogously, when $PB' \perp PC$ and $PC' \perp PA$). Thus, the proper triangle with minimal area is the pedal triangle $A'B'C'$ of $P$ to triangle $ABC$. This triangle is indeed similar to triangle $ABC$ : letting $\theta = \angle PAB$ be the Brocard angle, it is the image of triangle $ABC$ under a rotation through $\theta - 90°$, followed by a homothety of ratio $|\sin\theta|$.

To construct this triangle, first draw the circles $\{x | \angle BXA = \angle BCA + \angle CAB\}$ and $\{Y | \angle CYB = \angle CAB + \angle ABC\}$and let $P'$ be their point of intersection (different from $B$). We have $\angle AP'C = \angle ABC + \angle BCA$, and $\angle P'AB = 180° - \angle ABP' - \angle BP'A = 180° - (\angle ABC - \angle P'BC) - (\angle BCA + \angle CAB) = \angle P'BC$, and similarly $\angle P'BC = \angle P'CA$. Therefore $P = P'$. Finally, drop the perpendiculars from $P'$ to the sides of triangle $ABC$ to form $A'$, $B'$, $C'$. This completes the construction.

**3.** Let the incircle of triangle $ABD$ be tangent to $\overline{AD}$ at $T_1$ and let the incircle of triangle $ACD$ be tangent to $\overline{AD}$ at $T_2$ . Then

$$DT_1 = \frac{1}{2}(DA + DB - AB)$$

and $DT_2 = \frac{1}{2}(DA + DC - AC)$

First suppose a circle can be inscribed inside $AB_1DC_1$. Let it be tangent to sides $AB_1$, $B_1D$, $DC_1$, $C_1A$ at points $E$, $F$, $G$, $H$, respectively. Using equal tangents, we have

$AB - BD = (AH + HB) - (BF - DF)$

$= (AH + BF) - (BF - DF) = AH + DF$

and similarly $AC - CD = AE + DG$. Also, $AH + DF = AE + DG$ by equal tangents, implying that $AB - BD = AC - CD$ and thus $DA + DB - AB = DA + DC - AC$. Therefore $DT_1 = DT_2$, $T_1 = T_2$, and the two given incircles are tangent to each other. Next suppose the two incircles are tangent to each other. Then $DA + DB - AB = DA + DC - AC$. Let $\omega$ be the incircle of $ABB_1$, and let $D'$ be the point on $\overline{BB_1}$ (different from $B_1$) such that line $CD'$ is tangent to $\omega$. Suppose, by way of contradiction, that $D \neq D'$. From the result in the last paragraph, we know that the incircles of triangles $ABD'$ and $ACD'$ are tangent and hence $D'A + D'B - AB = D'A + D'C - AC$. Because $DB - AB = DC - AC$ and $D'B - AB = D'C - AC$, we must have $DB - D'B = DC = D'C$ by subtraction. Thus $DD' = |DB - D'B| = |DC - D'C|$. The triangle inequality then fails in triangle $DD'C$, a contradiction. This completes the proof.

**4.** Assume without loss of generality that $x = \max\{x, y, z\}$.

If $x \geq y \geq z$, then

$x^2y + y^2z + z^2x \leq x^2y + y^2z + z^2x + z(xy + (x - y)(y - z))$

$= (x+z)^2 y = 4\left(\frac{1}{2} - \frac{1}{2}y\right)\left(\frac{1}{2} - \frac{1}{2}y\right)y \leq \frac{4}{27}$, where the last inequality follows from the AM-GM inequality. Equality occurs if and only if $z = 0$ (from the first inequality) and $y = \frac{1}{3}$, in which case $(x, y, z) = \left(\frac{2}{3}, \frac{1}{3}, 0\right)$.

If $x \geq z \geq y$, then $x^2y + y^2z + z^2x = x^2z + z^2y + y^2x - (x - z)(z - y)(x - y)$

$\leq x^2z + z^2y + y^2x \leq \frac{4}{27}$

where the second inequality is true from the result we proved for $x \geq y \geq z$ (except with $y$ and $z$ reversed). Equality holds in the first inequality only when two of $x$, $y$, $z$ are equal, and in the second inequality only when $(x, z, y) = \left(\frac{2}{3}, \frac{1}{3}, 0\right)$. Because these conditions can't both be true, the inequality is actually strict in this case.

Therefore the inequality is indeed true, and equality holds when $(x, y, z)$ equals

$$\left(\frac{2}{3},\frac{1}{3},0\right),\left(\frac{1}{3},0,\frac{2}{3}\right), \text{ or } \left(0,\frac{2}{3},\frac{1}{3}\right)$$

5. We know that

$$m_a^2=\frac{1}{4}(2b^2+2c^2-a^2),\ m_b^2=\frac{1}{4}(2a^2+2c^2-b^2)$$

so $m_a^2-m_b^2=\frac{3}{4}(b^2-a^2)$

If the desired condition $m_a - m_b = b - a \neq 0$ is to be satisfied, then necessarily $m_a+m_b=\frac{3}{4}(b+a)$. Find the system of equations $m_a - m_b = b - a$

$$m_a+m_b=\frac{3}{4}(b+a)$$

we find that we would then have

$$m_a=\frac{1}{8}(7b-a), m_b=\frac{1}{8}(7a-b),$$

and $a+m_a=b+m_b=\frac{7}{8}(a+b)$. Thus $k=\frac{7}{8}$

Now we examine for what $a \neq b$ there exists a triangle $ABC$ with sides $a$, $b$ and medians $m_a=\frac{1}{8}(7b-a), m_b=\frac{1}{8}(7a-b)$. We can find all three side lengths of the triangle $AB_1G$, where $G$ is the centroid of the triangle $ABC$ and $B_1$ is the midpoint of the side $AC$:

$$AB_1=\frac{b}{2}, AG=\frac{2}{3}m_a=\frac{2}{3}\cdot\frac{1}{8}(7b-a)=\frac{1}{12}(7b-a)$$

$$B_1G=\frac{1}{3}m_b=\frac{1}{3}\cdot\frac{1}{8}(7a-b)=\frac{1}{24}(7a-b)$$

Examining the triangle inequalities for these three lengths, we get the condition $\frac{1}{3}<\frac{a}{b}<3$, from which the value $\frac{a}{b}=1$ has to be excluded by assumption. This condition is also sufficient: once the triangle $AB_1G$ has been constructed, it can always be completed to a triangle $ABC$ with $b = AC$, $m_a = AA_1$, $m_b = BB_1$. Then from the equality $m_a^2-m_b^2=\frac{3}{4}(b^2-a^2)$ we would also have $a = BC$.

6. Because $10^2 . 30^{2n}$ is even, $x$ and $y$ must have the same parity. Then $(x, y)$ is a valid solution if and only if $(u, v)=\left(\frac{x+y}{2},\frac{x-y}{2}\right)$ is a pair of positive integers that satisfies $u > v$ and $uv = 5^2 \cdot 30^{2n}$. Now $5^2 \cdot 30^{2n} = 2^{2n} . 3^{2n} . 5^{2n+2}$ has exactly $(2n + 1)^2(2n + 3)$ factors. Thus without the condition $u > v$ there are exactly $(2n + 1)^2(2n + 3)$ such pairs $(u, v)$. Exactly onepair has $u = v$, and by symmetry half of the remaining pairs have $u > v$. It follows that there are

$$\frac{1}{2}((2n+1)^2(2n+3)-1)=(n+1)(4n^2+6n+1)$$

valid pairs.

Now suppose that $(n + 1)(4n^2 + 6n + 1)$were a square. Because $n + 1$ and $4n^2 + 6n + 1 = (4n + 2)(n + 1) - 1$ are coprime, $4n^2 + 6n + 1$ must be a square as well. However, $(2n + 1)^2 < 4n^2 + 6n + 1 < (2n + 2)^2$, a contradiction.

7. **(a)** No matter what $x$ is, there is a unique value $\lambda=\frac{b-x}{b-a}$ such that $x = \lambda a + (1 - \lambda)b$. Also, $0<\frac{b-x}{b-a}<1 \Leftrightarrow a<x<b$, which proves the claim.

**(b)** The condition given says that $f$ is strictly convex. Stated geometrically, whenever $x < t < y$ the point $(t, f(t))$ lies strictly below the line joining $(x, f(x))$ and $(y, f(y))$. Suppose there were a parallelogram on the graph of $f$ whose vertices, from left to right, have $x$-coordiantes $a$, $b$, $d$, $c$. Then either $(b, f(d))$ or $(d, f(d))$ must lie on or above the line joining $(a, f(a))$ and $(c, f(c))$, a contradiction.

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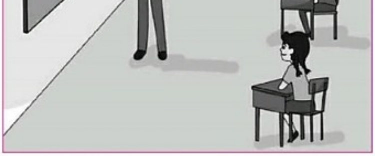
## COMIC CAPSULE


I know what your science teacher said, but this is one time when you CAN get something from nothing.
0! = 1

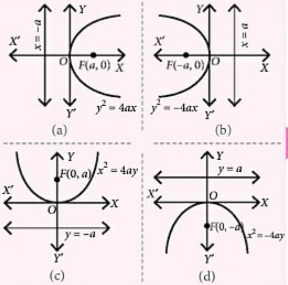
CONCEPT
MAP

# PARABOLA

Class XI

## Definition

Parabola is locus of point which moves in a plane such that its distance from a fixed point is equal to its distance from a fixed line. The fixed point is the focus. The fixed line is the directrix.



(a) (b) (c) (d)

## Equation of Tangent

| Curve | Point | Equation of Tangent |
|---|---|---|
| $y^2 = 4ax$ | $(x_1, y_1)$ | $yy_1 = 2a(x + x_1)$ (Point form) |
| | $(at^2, 2at)$ | $ty = x + at^2$ (Parametric form) |
| | $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ (slope is given) | $y = mx + \frac{a}{m}$ (Slope form) |

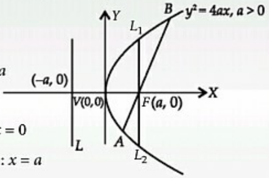
## Equation of Normal

| Curve | Point | Equation of Normal |
|---|---|---|
| $y^2 = 4ax$ | $(x_1, y_1)$ | $y - y_1 = -\frac{y_1}{2a}(x - x_1)$ (One point form) |
| | $(at^2, 2at)$ | $y + tx = at^3 + 2at$ (Parametric form) |
| | $(am^2, -2am)$ | $y = mx - 2am - am^3$ (Point slope form) |

## Standard Form

Standard form of parabola is $y^2 = 4ax$, $a > 0$

- Focus (F) : (a, 0)
- Vertex : (0, 0)
- Directrix (L) : $x = -a$
- Axis : $y = 0$
- Tangent at vertex : $x = 0$
- Latus Rectum, $L_1L_2 : x = a$
- Length of latus rectum = 4a

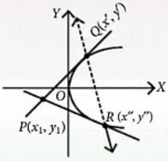


## Condition of Tangency of a Line to the Parabola

| Parabola | Line | Condition |
|---|---|---|
| $y^2 = 4ax$ | $y = mx + c$ | $c = \frac{a}{m}$ |
| | $x\cos\alpha + y\sin\alpha = p$ | $p = -a\tan\alpha\sin\alpha$ |
| | $lx + my + n = 0$ | $ln = am^2$ |

## Some Important Results

- Locus of point of intersection of perpendicular tangents to the parabola is its directrix, $x = -a$.
- Equation of chord of parabola bisected at point $(x_1, y_1)$ is given by $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$
- Equation of chord of contact (QR) of tangents to a parabola :
  - Equation of QR is given by $yy_1 = 2a(x + x_1)$
  - If $Q(at_1^2, 2at_1)$, $R(at_2^2, 2at_2)$ then equation of QR is given by $y(t_1 + t_2) = 2x + 2at_1t_2$



- The length of chord of the parabola $y^2 = 4ax$ passing through the vertex and making an angle θ with the axis is $4a\cot\theta\operatorname{cosec}\theta$.

CONCEPT MAP

# CONTINUITY

Class XII

## CONTINUITY

A function $f(x)$ is said to be continuous if its graph has no break or hole on it.

## Continuity of a Function at a Point

The real valued function $y = f(x)$ is said to be continuous at $x = a$ if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

## Continuity of a Function in an Interval

- $f(x)$ is said to be continuous in an open interval $(a, b)$ if it is continuous at each point of the interval.
- $f(x)$ is said to be continuous in the closed interval $[a, b]$ if $f(x)$ is continuous in $(a, b)$, $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$.

## Continuity Everywhere

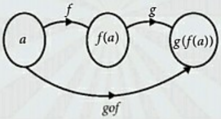
A function is said to be continuous everywhere if it is continuous on the entire real number line $(-\infty, \infty)$.

## Algebra of Continuous Functions

- If $f(x)$ is continuous, then $kf(x)$ is continuous for any constant $k$.
- If $f(x)$ and $g(x)$ are continuous, then $f(x) \pm g(x)$ and $f(x)g(x)$ are continuous in their domain. Also, $\frac{f(x)}{g(x)}$ is continuous at all values of $x$ for which $g(x) \neq 0$.
- If $f(x)$ is continuous, then $f^n(x)$, $\forall\, n \in N$ is continuous.

## Continuity of Composition of Two Functions

If $f$, $g$ are continuous functions, then $fog$ and $gof$ are continuous. If $f$ is continuous at a point $x = a$ and $g$ is continuous at $f(a)$, then $gof$ is continuous at $x = a$.



## DISCONTINUITY

The function $f(x)$ will be discontinuous at $x = a$ if it is not continuous at that point.

## Types of Discontinuity

- **Discontinuity of first kind (Jump Discontinuity) :** The function $f(x)$ is said to has discontinuity of first kind or jump discontinuity at $x = a$ if $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both exists finitely but are not equal.
- **Discontinuity of second kind (Oscillating Discontinuity) :** The function $f(x)$ is said to has discontinuity of second kind (oscillating discontinuity) at $x = a$ if $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both does not exist finitely or do not approach a definite value at $x = a$.
- **Removable Discontinuity :** The function $f(x)$ is said to has removable discontinuity at $x = a$ if $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both exists and are equal but not equal to $f(a)$.

## Continuity of Some Common Functions

| Function $f(x)$ | Interval in which $f(x)$ is continuous |
|---|---|
| Constant function : $f(x) = C$ | $R$ |
| Identity function : $f(x) = x$ | $R$ |
| Polynomial function :<br>$f(x) = a_0x^n + a_1x^{n-1} + .... + a_n$ | $R$ |
| Rational function :<br>$f(x)=\frac{p(x)}{q(x)};q(x)\neq 0$ | $R - \{x : q(x) = 0\}$ |
| Modulus function : $f(x) = \lvert x \rvert$ | $R$ |
| $x^{-n}$; $n$ is positive integer | $R - \{0\}$ |
| $\sin x$, $\cos x$, $e^x$ | $R$ |
| $\tan x$, $\sec x$ | $R-\{(2n+1)\frac{\pi}{2}:n\in Z\}$ |
| $\cot x$, $\mathrm{cosec}\, x$ | $R - \{ n\pi : n \in Z\}$ |
| $\log x$ | $(0, \infty)$ |

# YOU ASK
# WE ANSWER

**Do you have a question that you just can't get answered?**
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

**1.** If $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$, then find the value of $a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{(1-c^2)}$.

*(Prateush Sharma)*

**Ans.** Let $\sin^{-1}a = A \Rightarrow a = \sin A$ ...(i)

$\sin^{-1}b = B \Rightarrow b = \sin B$ ...(ii)

and $\sin^{-1}c = C \Rightarrow c = \sin C$ ...(iii)

$\therefore \quad \sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$

$\Rightarrow \quad A + B + C = \pi$ ...(iv)

Now, $a\sqrt{1-a^2} + b\sqrt{1-b^2} + c\sqrt{1-c^2}$

$$= \sin A\sqrt{1-\sin^2 A} + \sin B\sqrt{1-\sin^2 B} + \sin C\sqrt{1-\sin^2 C}$$
$$= \sin A \cos A + \sin B \cos B + \sin C \cos C$$
$$= \frac{\sin 2A + \sin 2B + \sin 2C}{2}$$
$$= \frac{2\sin(A+B)\cos(A-B) + \sin 2C}{2}$$
$$= \frac{2\sin(\pi - C)\cos(A-B) + 2\sin C\cos C}{2} \quad \text{[Using (iv)]}$$
$$= \frac{2\sin C\cos(A-B) + 2\sin C\cos C}{2}$$
$$= \frac{2\sin C\left[\cos(A-B) + \cos(\pi-(A+B))\right]}{2}$$
$$= \sin C(\cos(A-B) - \cos(A+B))$$
$$= 2\sin C \sin A \sin B = 2abc$$

**2.** Find the extreme values of the function $f(x) = [\cos x - 1] + [\cos 2x - 1] + \ldots\ldots + [\cos nx - 1]$, $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right], n \in I^+$.

*(Tina dabi, Chandigarh)*

**Ans.** We have,

$$f(x) = [\cos x - 1] + [\cos 2x - 1] + \ldots\ldots + [\cos nx - 1]$$
$$= [\cos x] - 1 + [\cos 2x] - 1 + \ldots\ldots + [\cos nx] - 1$$
$$= -n + [\cos x] + [\cos 2x] + \ldots\ldots + [\cos nx]$$

Thus, we have $f_{\max} = -n + n = 0$ at $x = 0$

and $f_{\min} = -n + (\underbrace{0-1+0+1}_{\text{these values now repeat themselves}} + \ldots\ldots)$ at $x = \frac{\pi}{2}$

*i.e.,* $f_{\min} = \begin{cases} -n, & \text{if } n = 4k \text{ or } 4k+1 \\ -n-1, & \text{if } n = 4k+2 \text{ or } 4k+3 \end{cases}$

Hence, the extreme values of $f(x)$ are $-n, -n-1, 0$.

**3.** $\bar{b}z + b\bar{z} = c$ is the equation of a straight line. If $z_1$ and $z_2$ be mirror image of each other in this line, then prove that $\bar{b}z_2 + b\bar{z}_1 = c$.

*(Rituraj, Madhya Pradesh)*

**Ans.** If $z_1$ and $z_2$ are mirror image of each other in the line

$\bar{b}z + b\bar{z} = c$ ...(i)

then the mid-point of $z_1, z_2$ must lie on (i) and the line joining $z_1$, $z_2$ must be perpendicular to (i).

$$\therefore \quad \bar{b}\left(\frac{z_1+z_2}{2}\right) + b\overline{\left(\frac{z_1+z_2}{2}\right)} = c$$

*i.e.,* $\bar{b}(z_1+z_2) + b(\bar{z}_1 + \bar{z}_2) = 2c$

*i.e.,* $(\bar{b}z_1 + b\bar{z}_2) + (\bar{b}z_2 + b\bar{z}_1) = 2c$ ...(ii)

Putting $z = x + iy$ in equation (i), it reduces to

$\bar{b}(x+iy) + b(x-iy) - c = 0$

*i.e.,* $(\bar{b}+b)x + i(\bar{b}-b)y - c = 0$

whose slope $= \dfrac{(b+\bar{b})}{i(b-\bar{b})}$

Equation of the line joining $z_1, z_2$ is

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0$$

*i.e.,* $z\bar{a} - \bar{z}a + d = 0$, where $a = z_1 - z_2$

and $d = z_1\bar{z}_2 - \bar{z}_1 z_2$

Putting $z = x + iy$, the above equation reduces to

$\bar{a}(x+iy) - a(x-iy) + d = 0$

*i.e.,* $(\bar{a}-a)x + i(\bar{a}+a)y + d = 0$

whose slope $= \dfrac{(a-\bar{a})}{i(a+\bar{a})}$

Now, $\dfrac{(b+\bar{b})}{i(b-\bar{b})} \times \dfrac{(a-\bar{a})}{i(a+\bar{a})} = -1$

$\Rightarrow \quad (b+\bar{b})(a-\bar{a}) = (b-\bar{b})(a+\bar{a}) \Rightarrow \bar{b}a = b\bar{a}$

$\Rightarrow \quad \bar{b}(z_1 - z_2) = b(\bar{z}_1 - \bar{z}_2)$

$\Rightarrow \quad \bar{b}z_1 + b\bar{z}_2 = \bar{b}z_2 + b\bar{z}_1$ ...(iii)

From equations (ii) and (iii), we have

$\bar{b}z_2 + b\bar{z}_1 = c$,

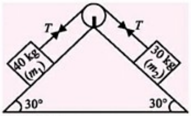
which is the desired result.

◆◆

# FULL LENGTH PRACTICE PAPER
# BITSAT

## SECTION-I (PHYSICS)

**1.** Two masses 40 kg and 30 kg are connected by a weightless string passing over a frictionless pulley as shown in the figure.



The tension in the string will be
(a) 188 N (b) 368 N (c) 288 N (d) 168 N

**2.** The half-life of $^{60}$Co is approximately 5.25 years. In a sample containing 1 g of freshly prepared $^{60}$Co, how much of the isotope will be left after 21 years?
(a) 125 mg (b) 62.5 mg
(c) 31.25 mg (d) nothing will be left

**3.** At constant temperature, the volume of a gas is to be decreased by 4%. The pressure must be increased by
(a) 5.34% (b) 4.16% (c) 2.96% (d) 3.86%

**4.** The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 $\mu$T. What will be its value at the centre of the loop?
(a) 250 $\mu$T (b) 150 $\mu$T (c) 125 $\mu$T (d) 75 $\mu$T

**5.** The input resistance of a silicon transistor is 100 $\Omega$. Base current is changed by 40 $\mu$A which results in a change in collector currently by 2 mA. This transistor is used as a common emitter amplifier with a load resistance of 4 k$\Omega$. The voltage gain of the amplifier is
(a) 2000 (b) 3000 (c) 4000 (d) 1000

**6.** An alternating emf $e = 220\sqrt{2}\sin 100t$ V is applied to a capacitor 1 $\mu$F. The current flowing through the capacitor is
(a) 22 mA (b) 12 mA (c) 32 mA (d) 42 mA

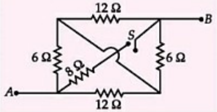
**7.** In a race for 100 m dash, the first and the second runners have a gap of one metre at the mid way stage. Assuming the first runner goes steady, by what percentage should the second runner increase his speed just to win the race?
(a) 2% (b) 4%
(c) more than 4% (d) less than 4%

**8.** Two linear SHMs of equal amplitude $A$ and angular frequencies $\omega$ and $2\omega$ are impressed on a particle along the axes $x$ and $y$ respectively. If the initial phase difference between them is $\pi/2$, the resultant path followed by the particle is
(a) $y^2 = x^2(1 - x^2/A^2)$ (b) $y^2 = 2x^2(1 - x^2/A^2)$
(c) $y^2 = 4x^2(1 - x^2/A^2)$ (d) $y^2 = 8x^2(1 - x^2/A^2)$

**9.** A calorie is a unit of heat and equals 4.2 J. Suppose we employ a system of units in which the unit of mass is $\alpha$ kg, the unit of length is $\beta$ m and the unit of time is $\gamma$ s. In this new system, 1 calorie will be
(a) $\alpha^{-1}\beta^{-2}\gamma^2$ (b) $4.2\alpha\beta^2\gamma^2$
(c) $\alpha\beta^2\gamma^2$ (d) $4.2\alpha^{-1}\beta^{-2}\gamma^2$

**10.** According to Bohr's theory the energy of an electron in hydrogen atom starts corresponding to $n = 1, 2, 3, 4$ is $-$ 13.6 eV, $-$ 3.4 eV, $-$ 1.51 eV and $-$0.85 eV respectively. In order to obtain emission of $H_\beta$ line of Balmer series, the energy that has to be given to a normal hydrogen atom is
(a) 12.75 eV (b) 14.45 eV
(c) 2.55 eV (d) 4.25 eV

**11.** The equivalent resistance between points $A$ and $B$ with switch $S$ open and closed are respectively



(a) 4 $\Omega$, 8 $\Omega$ (b) 8 $\Omega$, 4 $\Omega$
(c) 6 $\Omega$, 9 $\Omega$ (d) 9 $\Omega$, 6 $\Omega$

**12.** A sand bag of mass $m$ is suspended from a long string. A bullet of mass $\frac{m}{20}$ moving with a horizontal velocity $v$ strikes it and gets embedded into it. Calculate the velocity gained by the bag in this process and fraction of energy lost in the process.

(a) $\frac{v}{21}, \frac{20}{21}$ (b) $\frac{v}{24}, \frac{18}{20}$ (c) $\frac{v}{21}, \frac{20}{22}$ (d) $\frac{v}{19}, \frac{20}{21}$

**13.** Consider an $n$-$p$-$n$ transistor amplifier in common emitter configuration. The current gain of the transistor is 100. If the collector current changes by 1 mA, what will be the change in emitter current ?
(a) 1.1 mA (b) 1.01 mA (c) 0.01 mA (d) 10 mA

**14.** When radiation of the wavelength $\lambda$ is incident on a metallic surface, the stopping potential is 4.8 V. If the same surface is illuminated with radiation of double the wavelength, then the stopping potential becomes 1.6 V. Then the threshold wavelength for the surface is
(a) $2\lambda$ (b) $4\lambda$ (c) $6\lambda$ (d) $8\lambda$

**15.** The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectrons emission from this substance is approximately
(a) 540 nm (b) 400 nm (c) 310 nm (d) 220 nm

**16.** The potential difference between the plates of a parallel plate capacitor is changing at the rate of $10^6$ V s$^{-1}$. If the capacitance is 2 μF, the displacement current in the dielectric of the capacitor will be
(a) 1 A (b) 2 A (c) 3 A (d) 4 A

**17.** In an experiment with sonometer, a tuning fork of frequency 256 Hz resonates with a length of 25 cm and another tuning fork resonates with a length of 16 cm. Tension in the string remains constant, the frequency of the second tuning fork is
(a) 204 Hz (b) 160 Hz (c) 400 Hz (d) 320 Hz

**18.** The maximum range of projectile fired with some initial velocity is found to be 1000 m, in the absence of wind and air resistance. The maximum height reached by the projectile is
(a) 250 m (b) 500 m (c) 1000 m (d) 2000 m

**19.** A physical quantity $X$ is given by $X = \frac{2k^3l^2}{m\sqrt{n}}$
The percentage error in the measurements of $k$, $l$, $m$ and $n$ are 1%, 2%, 3% and 4% respectively. The value of $X$ is uncertain by
(a) 8% (b) 10%
(c) 12% (d) none of these

**20.** A conducting metal circular-wire-loop of radius $r$ is placed perpendicular to a magnetic field which varies with time as $B = B_0 e^{-\frac{t}{\tau}}$, where $B_0$ and $\tau$ are constants, at time $t = 0$. If the resistance of the loop is $R$ then the heat generated in the loop after a long time $(t \to \infty)$ is
(a) $\frac{\pi^2 r^4 B_0^4}{2\tau R}$ (b) $\frac{\pi^2 r^4 B_0^2}{2\tau R}$
(c) $\frac{\pi^2 r^4 B_0^2 R}{\tau}$ (d) $\frac{\pi^2 r^4 B_0^2}{\tau R}$

**21.** The cylindrical tube of spray pump has a cross-section of 8 cm$^2$, one end of which has 40 fine holes each of area $10^{-8}$ m$^2$. If the liquid flows inside the tube with a speed of 0.15 m min$^{-1}$, the speed with which the liquid is ejected through the holes is
(a) 50 m s$^{-1}$ (b) 5 m s$^{-1}$
(c) 0.05 m s$^{-1}$ (d) 0.5 m s$^{-1}$

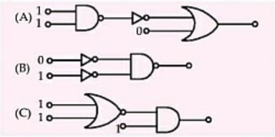
**22.** A body is fired vertically upwards. At half the maximum height, the velocity of the body is 10 m s$^{-1}$. The maximum height raised by the body is (Take $g = 10$ m s$^{-2}$)
(a) 5 m (b) 10 m (c) 15 m (d) 20 m

**23.** A satellite in a circular orbit of radius $R$ has a period of 4 hours. Another satellite with orbital radius $3R$ around the same planet will have a period (in hours)
(a) 16 (b) 4 (c) $4\sqrt{27}$ (d) $4\sqrt{8}$

**24.** A body of mass $M$ suspended from two springs separately executes simple harmonic motion. During oscillation the maximum velocity is equal in both cases. The ratio of amplitude $\frac{A_1}{A_2}$ is
(a) $\frac{k_1}{k_2}$ (b) $\frac{k_2}{k_1}$ (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_1^2}{k_2^2}$

**25.** Three point charges $+q$, $-2q$ and $+q$ are placed at points $(x = 0, y = a, z = 0)$, $(x = 0, y = 0, z = 0)$ and $(x = a, y = 0, z = 0)$ respectively. The magnitude and direction of the electric dipole moment vector of this charge assembly are
(a) $\sqrt{2}\,qa$ along the line joining the points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$.
(b) $qa$ along the line joining the points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$.
(c) $\sqrt{2}\,qa$ along $+x$ direction
(d) $\sqrt{2}\,qa$ along $+y$ direction.

**26.** In the following combinations of logic gates, the outputs of A, B and C are respectively



(a) 0, 1, 1 (b) 0, 1, 0 (c) 1, 1, 0 (d) 1, 0, 1

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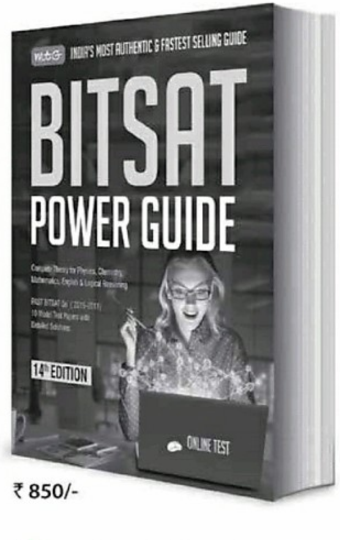

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**27.** A wire of length 100 cm is connected to a cell of emf 2 V and negligible internal resistance. The resistance of the wire is 3 Ω. The additional resistance required to produce a potential difference of 1 millivolt per cm is
(a) 60 Ω (b) 47 Ω (c) 57 Ω (d) 35 Ω

**28.** Two slits separated by a distance of 1 mm are illuminated with red light of wavelength $6.5 \times 10^{-7}$ m. The interference fringes are observed on a screen placed 1 m from the slits. The distance between the third dark fringe and the fifth bright fringe is equal to
(a) 0.65 mm (b) 1.63 mm
(c) 3.25 mm (d) 4.88 mm

**29.** A metal rod of Young's modulus $Y$ and coefficient of thermal expansion $\alpha$ is held at its two ends such that its length remains invariant. If its temperature is raised by $t$ °C, the linear stress developed in it is
(a) $\dfrac{\alpha t}{Y}$ (b) $\dfrac{Y}{\alpha t}$ (c) $Y\alpha t$ (d) $\dfrac{1}{(Y\alpha t)}$

**30.** When a dc voltage of 200 V is applied to a coil of self inductance $(2\sqrt{3}/\pi)$ H, a current of 1 A flows through it. But by replacing dc source with ac source of 200 V, the current in the coil is reduced to 0.5 A. Then the frequency of ac supply is
(a) 100 Hz (b) 75 Hz (c) 60 Hz (d) 50 Hz

**31.** A compound microscope has an eye piece of focal length 10 cm and an objective of focal length 4 cm. Find the magnification, if an object is kept at a distance of 5 cm from the objective, so that the final image is formed at the least distance of distinct vision 20 cm.
(a) 12 (b) 11 (c) 10 (d) 13

**32.** The angular momentum of an electron in the hydrogen atom is $3h/2\pi$. Here, $h$ is Planck's constant. The kinetic energy of this electron is
(a) 4.35 eV (b) 1.51 eV (c) 3.4 eV (d) 6.8 eV

**33.** If the length of stretched string is shortened by 40% and the tension is increased by 44%, then the ratio of the final and initial fundamental frequencies is
(a) 2 : 1 (b) 3 : 2 (c) 3 : 4 (d) 1 : 3

**34.** A uniform thin bar of mass $6m$ and length $12L$ is bent to make a regular hexagon. Its moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is
(a) $20mL^2$ (b) $6mL^2$ (c) $\dfrac{12}{5}mL^2$ (d) $30mL^2$

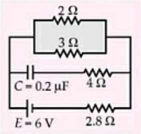
**35.** A solenoid of length 30 cm with 10 turns per centimetre and area of cross-section 40 $\text{cm}^2$ completely surrounds another co-axial solenoid of same length, area of cross-section 20 $\text{cm}^2$ with 40 turns per centimeter. The mutual inductance of the system is
(a) 10 H (b) 8 H (c) 3 mH (d) 30 mH

**36.** An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})\text{m}$ to $\vec{r}_2 = (4\hat{i} + 6\hat{j})\text{m}$ under a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})\text{N}$. The work done by this force is
(a) 63 J (b) 73 J (c) 83 J (d) 93 J

**37.** A prism of refractive index μ and angle $A$ is placed in the minimum deviation position. If the angle of minimum deviation is $A$, then the value of $A$ in terms of μ is
(a) $\sin^{-1}\left(\dfrac{\mu}{2}\right)$ (b) $\sin^{-1}\left(\sqrt{\dfrac{\mu-1}{2}}\right)$
(c) $2\cos^{-1}\left(\dfrac{\mu}{2}\right)$ (d) $\cos^{-1}\left(\dfrac{\mu}{2}\right)$

**38.** A launching vehicle carrying an artificial satellite of mass $m$ is set for launch on the surface of the earth of mass $M$ and radius $R$. If the satellite is intended to move in a circular orbit of radius $7R$, the minimum energy required to be spent by the launching vehicle on the satellite is (Gravitational constant = $G$)
(a) $\dfrac{GMm}{R}$ (b) $\dfrac{13GMm}{14R}$
(c) $\dfrac{GMm}{7R}$ (d) $\dfrac{GMm}{14R}$

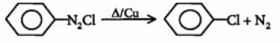
**39.** In the circuit shown, the internal resistance of the cell is negligible. The steady state current in the 2 Ω resistor is
(a) 0.6 A (b) 0.9 A
(c) 1.2 A (d) 1.5 A



**40.** Interference fringes were produced in Young's double slit experiment using light of wavelength 5000 Å. When a film of material $2.5 \times 10^{-3}$ cm thick was placed over one of the slits, the fringe pattern shifted by a distance equal to 20 fringe widths. The refractive index of the material of the film is
(a) 1.25 (b) 1.33 (c) 1.4 (d) 1.5

## SECTION-II (CHEMISTRY)

**41.** The correct decreasing order of acidic character is
(a) HClO > HBrO > HIO
(b) HIO > HBrO > HClO
(c) HBrO > HIO > HClO
(d) HClO > HIO > HBrO

**42.** 

(*A*)

Half-life is independent of concentration of *A*. After 10 minutes volume of $N_2$ gas is 10 L and after complete reaction 50 L. Hence, rate constant is

(a) $\frac{2.303}{10}\log 5\ \text{min}^{-1}$ (b) $\frac{2.303}{10}\log 1.25\ \text{min}^{-1}$

(c) $\frac{2.303}{10}\log 2\ \text{min}^{-1}$ (d) $\frac{2.303}{10}\log 4\ \text{min}^{-1}$

**43.** Reduction potentials of some ions are given below. Arrange them in decreasing order of oxidising power.

| **Ion** | **$ClO_4^-$** | **$IO_4^-$** | **$BrO_4^-$** |
|---|---|---|---|
| Reduction potential ($E°$) | 1.19 V | 1.65 V | 1.74 V |

(a) $ClO_4^- > IO_4^- > BrO_4^-$ (b) $IO_4^- > BrO_4^- > ClO_4^-$

(c) $BrO_4^- > IO_4^- > ClO_4^-$ (d) $BrO_4^- > ClO_4^- > IO_4^-$

**44.** The rapid change of pH near the stoichiometric point of an acid-base titration is the basis of indicator detection. pH of the solution is related to the ratio of the concentration of conjugate acid (HIn) and base ($In^-$) forms of the indicator by the expression

(a) $\log\frac{[In^-]}{[HIn]} = pK_a + pH$

(b) $\log\frac{[HIn]}{[In^-]} = K_a + pH$

(c) $\log\frac{[HIn]}{[In^-]} = pH - pK_a$

(d) $\log\frac{[In^-]}{[HIn]} = pH - pK_a$

**45.** The coagulation of 100 mL of colloidal solution of gold is completely prevented by addition of 0.25 g of a substance *X* to it before adding 1 mL of 10% NaCl solution. The gold number of *X* is

(a) 0.25 (b) 25 (c) 250 (d) 2.5

**46.** Consider the following statements :

1. Atomic hydrogen is obtained by passing hydrogen through an electric arc.
2. Hydrogen gas will not reduce heated aluminium oxide.
3. Finely divided palladium absorbs large volume of hydrogen gas.
4. Pure nascent hydrogen is best obtained by reacting Na with $C_2H_5OH$.

Which of the given statements is/are correct?

(a) 1 only (b) 2 only

(c) 1, 2 and 3 (d) 2, 3 and 4

**47.** The emf of the three galvanic cells given below are represented by $E_1$, $E_2$ and $E_3$.

I. $Zn \mid Zn^{2+}\ (1\ M) \parallel Cu^{2+}\ (1\ M) \mid Cu$
II. $Zn \mid Zn^{2+}\ (0.1\ M) \parallel Cu^{2+}\ (1\ M) \mid Cu$
III. $Zn \mid Zn^{2+}\ (1\ M) \parallel Cu^{2+}\ (0.1\ M) \mid Cu$

Which of the following is true?

(a) $E_1 > E_2 > E_3$ (b) $E_3 > E_2 > E_1$

(c) $E_3 > E_1 > E_2$ (d) $E_2 > E_1 > E_3$

**48.** Which of the following statements is correct regarding the ions, $Zn^{2+}$, $Ni^{2+}$ and $Cr^{3+}$ ?

(At. nos. Zn = 30, Ni = 28, Cr = 24)

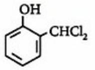
(a) Only $Zn^{2+}$ is colourless, $Ni^{2+}$ and $Cr^{3+}$ are coloured.
(b) All three are colourless.
(c) All three are coloured.
(d) Only $Ni^{2+}$ is coloured, $Zn^{2+}$ and $Cr^{3+}$ are colourless.

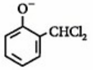
**49.** An element with molar mass $2.7 \times 10^{-2}$ kg mol$^{-1}$ forms a cubic unit cell with edge length 405 pm. If its density is $2.7 \times 10^3$ kg m$^{-3}$, what is the nature of the cubic unit cell?

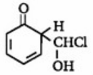
(a) Simple cubic (b) Face-centred

(c) Body-centred (d) End-centred

**50.** When phenol is reacted with $CHCl_3$ and NaOH followed by acidification, salicylaldehyde is obtained. Which of the following species are involved in the above mentioned reaction as intermediate?

(a) 

(b) 

(c) 

(d) None of these

**51.** The percentage of hydrogen in $H_2O$ and hydrogen peroxide are respectively 11.2% and 5.94%. This illustrates

(a) law of multiple proportions
(b) conservation of mass
(c) law of definite proportions
(d) law of reciprocal proportions.

**52.** Which of the following statements is not true?

(a) London smog is a mixture of smoke and fog.
(b) London smog is oxidising in nature.
(c) Photochemical smog causes irritation in eyes.
(d) Photochemical smog results in the formation of PAN.

**53.** $N_2$ and $O_2$ are converted into monocations, $N_2^+$ and $O_2^+$ respectively. Which of the following is wrong?

(a) In $N_2^+$, N – N bond weakens.
(b) In $O_2^+$, the O – O bond order increases.
(c) In $O_2^+$, paramagnetism decreases.
(d) $N_2^+$ becomes diamagnetic.

**54.** The two isomers *X* and *Y* with the formula $Cr(H_2O)_5ClBr_2$ were taken for experiment on depression in freezing point. It was found that one mole of *X* gave depression corresponding to 2 moles of particles and one mole of *Y* gave depression due to 3 moles of particles. The structural formulae of *X* and *Y* respectively are
(a) $[Cr(H_2O)_5Cl]Br_2$; $[Cr(H_2O)_4Br_2]Cl.\ H_2O$
(b) $[Cr(H_2O)_5Cl]Br_2$; $[Cr(H_2O)_3ClBr_2].\ 2H_2O$
(c) $[Cr(H_2O)_5Br]BrCl$; $[Cr(H_2O)_4ClBr]Br.\ H_2O$
(d) $[Cr(H_2O)_4Br_2]Cl.H_2O$; $[Cr(H_2O)_5Cl]Br_2$

**55.** Energy of an electron in hydrogen atom is given by $E = -\frac{13.6}{n^2}$ eV. Which one of the following statements is true if *n* is changed from 1 to 3? Energy will
(a) decrease three times (b) increase three times
(c) increase nine times (d) decrease nine times.

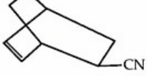
**56.** Two liquids *A* and *B* have vapour pressures in the ratio $p^\circ_A : p^\circ_B = 1 : 2$ at a certain temperature. Suppose that we have an ideal solutions of *A* and *B* in the mole fractions ratio $A : B = 1 : 2$, the mole fraction of *A* in the vapour in equilibrium with the solution at the given temperature is
(a) 0.25 (b) 0.2 (c) 0.5 (d) 0.33

**57.** 1500 mL flask contains 400 mg $O_2$ and 60 mg $H_2$ at 100°C. What is the total pressure in the flask?
(a) 0.66 atm (b) 0.867 atm
(c) 8.67 atm (d) 13.47 atm

**58.** In the reaction, $2X + B_2H_6 \longrightarrow [BH_2(X_2)]^+[BH_4]^-$
'*X*' cannot be
(a) $NH_3$ (b) $CH_3NH_2$
(c) $(CH)_3)_2NH$ (d) $(CH_3)_3N$

**59.** A racemic mixture is a mixture of
(a) *meso* and its isomers
(b) *d*- and its *l*-isomers in equal proportions
(c) *d*- and its *l*- isomers in different proportions
(d) *meso* and *d*-isomers.

**60.** The Diels—Alder reaction between 1,3-cyclohexadiene and acrylonitrile gives the adduct,



Its IUPAC name is
(a) bicyclo [2.2.2] oct-2-en-5-nitrile
(b) bicyclo [2.2.2] oct-5-en-2-carbonitrile
(c) 3-cyano bicyclo [2.2.2] oct-5-ene
(d) 2-cyano bicyclo [2.2.2] oct-5-ene.

**61.** Consider the following haloalkanes :
(1) $CH_3I$ (2) $CH_3F$ (3) $CH_3Cl$ (4) $CH_3Br$
The correct sequence of increasing order of dipole moments is
(a) $1 < 2 < 3 < 4$ (b) $4 < 3 < 2 < 1$
(c) $1 < 4 < 2 < 3$ (d) $3 < 4 < 1 < 2$

**62.** Identify '*Z*' in the given reaction.

$$C_6H_5NH_2 \xrightarrow{(CH_3CO)_2O} X \xrightarrow{Br_2/CCl_4} Y \xrightarrow[H^+]{HOH} Z$$

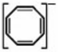
(a) *p*-Bromoaniline
(b) *p*-Bromoacetophenone
(c) *p*-Bromoacetanilide (d) *o*-Bromoacetanilide

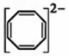
**63.** What will be Δ*G* for the reaction at 25°C when partial pressures of reactants $H_2$, $CO_2$, $H_2O$ and CO are 10, 20, 0.02 and 0.01 atm respectively. (Given : $G^\circ_{H_2O} = -228.58$ kJ, $G^\circ_{CO} = -137.15$ kJ, $G^\circ_{CO_2} = -394.37$ kJ.)
(a) +5.61 kJ (b) –5.61 kJ
(c) 7.09 kJ (d) –8.13 kJ

**64.** Which among the following is aromatic?

(a)

(b)

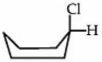
(c)


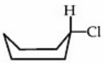
(d)


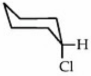
**65.** In the reaction : $I_2 + 2S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$
(a) $I_2$ is reducing agent.
(b) $I_2$ is oxidising agent and $S_2O_3^{2-}$ is reducing agent.
(c) $S_2O_3^{2-}$ is oxidising agent.
(d) $I_2$ is reducing agent and $S_2O_3^{2-}$ is oxidising agent.

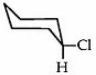
**66.** An ester (*A*) with molecular formula, $C_9H_{10}O_2$ was treated with excess of $CH_3MgBr$ and the complex so formed was treated with $H_2SO_4$ to give an olefin (*B*). Ozonolysis of (*B*) gave a ketone with molecular formula $C_8H_8O$ which shows positive iodoform test. The structure of (*A*) is
(a) $C_6H_5COOC_2H_5$ (b) $C_6H_5COOC_6H_5$
(c) $CH_3OCH_2COC_6H_5$
(d) *p*-$CH_3OCOC_6H_4COCH_3$

**67.** The most stable conformation of chlorocyclohexane at room temperature is

(a)


(b)


(c)


(d)


**68.** If $M$ is the element of actinide series, the degree of complex formation decreases in the order
(a) $M^{4+} > M^{3+} > MO_2^{2+} > MO_2^{+}$
(b) $MO_2^{+} > MO_2^{2+} > M^{3+} > M^{4+}$
(c) $M^{4+} > MO_2^{2+} > M^{3+} > MO_2{}^{+}$
(d) $MO_2^{2+} > MO_2{}^{+} > M^{4+} > M^{3+}$

**69.** The value of p$K_w$ of water
(a) increases with increase in temperature
(b) decreases with rise in temperature
(c) does not change with variation in temperature
(d) increases till 50°C and there after decreases.

**70.** A red coloured oxide ($X$) on treatment with conc. $HNO_3$ gives a compound ($Y$). ($Y$) with HCl produces a chloride ($Z$) which is insoluble in cold water but soluble in hot water. ($Z$) can also be formed by treating ($X$) with conc. HCl. Compounds $X$, $Y$ and $Z$ are
(a) $Pb_3O_4$, $PbO_2$, $PbCl_2$ (b) $Mn_3O_4$, $MnO_2$, $MnCl_2$
(c) $Fe_3O_4$, $Fe_2O_3$, $FeCl_3$ (d) $Fe_3O_4$, FeO, $FeCl_2$

**71.** The quantum numbers +1/2 and –1/2 for the electron spin represent
(a) rotation of the electron in clockwise and anticlockwise direction respectively
(b) rotation of the electron in anticlockwise and clockwise direction respectively
(c) magnetic moment of the electron pointing up and down respectively
(d) two quantum mechanical spin states which have no classical analogue.

**72.** 500 mL of a hydrocarbon gas burnt in excess of oxygen yielded 2500 mL of $CO_2$ and 3.0 L of water vapour. The formula of the hydrocarbon is
(a) $C_5H_{12}$ (b) $CH_4$ (c) $C_2H_4$ (d) $C_3H_6$

**73.** Which of the following is most reactive towards electrophilic substitution?
(a) Nitrobenzene (b) Aniline
(c) Aniline hydrochloride
(d) $N$-Acetylaniline

**74.** Antihistamines are not helpful
(a) in curing nasal allergies
(b) in treating rashes caused by itching
(c) in bringing down acute fever
(d) in vasodilation.

**75.** Match the column I with column II and mark the appropriate choice.

| **Column I (Property)** | | **Column II (Metal)** | |
|---|---|---|---|
| (A) | Element with highest second ionisation enthalpy | (i) | Cr |
| (B) | Element with highest third ionisation enthalpy | (ii) | Cu |
| (C) | $M$ in $M(CO)_6$ is | (iii) | Zn |
| (D) | Element with highest heat of atomisation | (iv) | Ni |

(a) (A) → (ii), (B) → (iii), (C) → (i), (D) → (iv)
(b) (A) → (iv), (B) → (iii), (C) → (i), (D) → (ii)
(c) (A) → (iii), (B) → (i), (C) → (ii), (D) → (iv)
(d) (A) → (i), (B) → (ii), (C) → (iii), (D) → (iv)

**76.** Which of the following is not a use of graphite?
(a) For electrodes in batteries.
(b) Crucibles made from graphite are used for its inertness to dilute acids and alkalies.
(c) For adsorbing poisonous gases.
(d) Lubricant at high temperature.

**77.** *cis*-Polyisoprene possesses elastic property because
(a) it is soft and soluble in non-polar solvent
(b) it is unsaturated and porous
(c) it has a coiled structure and chains held together by weak van der Waals forces
(d) it has a fibrous structure and reactive sites at various double bonds.

**78.** The number of $sp^3 - s$, $sp^2 - s$, $sp^3 - sp^2$ and $sp^2 - sp^2$ bonds present in 2-butene respectively, are
(a) 2, 1, 1, 4 (b) 6, 2, 2, 1
(c) 2, 3, 4, 6 (d) 2, 2, 6, 1

**79.** Identify the correct statement for change in Gibbs' energy for a system ($\Delta G_{system}$) at constant temperature and pressure.
(a) If $\Delta G_{system} = 0$, the system has attained equilibrium.
(b) If $\Delta G_{system} = 0$, the system is still moving in a particular direction.
(c) If $\Delta G_{system} = 0$, the process is not spontaneous.
(d) If $\Delta G_{system} = 0$, the process is spontaneous.

**80.** For which of the following amino acids, van-Slyke estimation method is not applicable?
(a) Alanine (b) Aspartic acid
(c) Serine (d) Proline

## SECTION-III (ENGLISH AND LOGICAL REASONING)

**Directions (Q. 81–83) :** Read the passage and answer the following questions.

In the past thirty years, drugs have been discovered that prevent and cure physical disease and reverse the disturbances that occur in certain mental illness.

Excitement over what drugs can do has led people to believe that any ailment, infective or psychic, can be relieved by taking a pill. At the first sign of nervousness, they try pep-up pills. Medical journals now advertise tranquillizers, and other mood-altering drugs; doctors prescribe them; and the public expect miracles from them. In such an atmosphere, it is not surprising that drug abuse has spread.

**81.** According to the author, in recent years there has been ______ .
(a) recognition of the ill-effects of medicine
(b) a misplaced trust in drugs
(c) a distrust of drugs (d) none of these

**82.** According to the passage, the medicines that have been discovered in recent times ______ .
(a) can reduce mental illnesses
(b) cannot cure mental illnesses
(c) can cure mental illnesses
(d) can help treat some symptoms of mental illnesses

**83.** People often believe that ______ .
(a) medicines can cure all the diseases
(b) doctors cannot cure all the diseases
(c) medicines cannot cure all the diseases
(d) doctors can cure all the diseases

**Directions : (Q. 84–86) :** Choose the incorrect word in the given sentence.

**84.** My father is in the bad mood today.
(a) father (b) the (c) mood (d) today

**85.** The doctor attended to the patient very quietly.
(a) attended (b) the
(c) very (d) no error

**86.** Supposing if you are arrested, what will you do?
(a) Supposing (b) arrested
(c) will (d) do

**Directions (Q. 87–89) :** Choose the antonym for the given words from each set.

**87.** Punctilious
(a) Careless (b) Curious
(c) Irregular (d) Fussy

**88.** Capacious
(a) Caring (b) Limited
(c) Changeable (d) Foolish

**89.** Bizarre
(a) Gentle (b) Same (c) Soft (d) Usual

**Directions (Q. 90–91) :** Choose the synonym for the given words from each set.

**90.** Diligent
(a) Modest (b) Slack
(c) Industrious (d) Intelligent

**91.** Vindicate
(a) Convict (b) Fault (c) Judge (d) Justify

**Directions (Q. 92–93) :** Pick up from answers choice which will complete the sentences correctly.

**92.** The twins are so alike that I cannot ........ one from the other.
(a) say (b) notice (c) discern (d) tell

**93.** The chief guest's speech was so ........ that I was all ears.
(a) interesting (b) indeterminate
(c) intricate (d) interminable

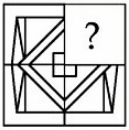
**Directions : (Q. 94–95) :** Select the pair of words/word, which are related in the same way as the capitalised words are related to each other.

**94.** INDOLENCE : WORK :: TACITURN : ?
(a) cheat (b) act (c) speak (d) observe

**95.** SCALES : FISH :: ?
(a) Bear : Fur (b) Woman : Dress
(c) Skin : Man (d) Tree : Leaves

**96.** A, B, C, D, E, F and G are standing in a queue not necessarily in that order. Each one is wearing a cap of different colour: violet, indigo, blue, green, yellow, orange and red. D is able to see in front of him green and blue but not violet. F can see violet and yellow but not red. G can see caps of all the colours other than orange. If E is wearing indigo then F wears
(a) Blue (b) Red (c) Violet (d) Orange

**97.** Select a figure from the options which completes the question figure shown below.



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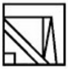
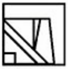


(b)


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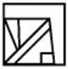
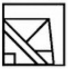
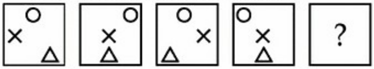
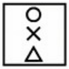
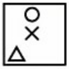


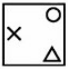
(d)


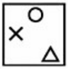
**98.** Find the figure that comes in place of '?' and follows/continues the same relationship.



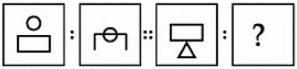
(a) 

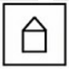
(b) 

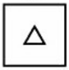
(c) 

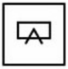
(d) 

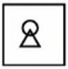
**99.** If figure on the left of ': :' follows a particular relationship, establish the same relationship on the right as well.



(a) 

(b) 

(c) 

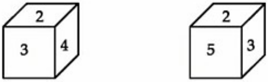
(d) 

**100.** If 23496 = CYSTE and 1578 = BURP, then 85367 = ?

(a) PYSER (b) PUYER

(c) PUYSR (d) PSERY

**101.** Two positions of a dice is shown below. When 4 is at the bottom, what number will be at the top?

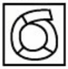


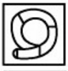
(a) 1 (b) 3 (c) 4 (d) 5

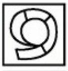
**102.** Laxman went 15 km to the West from my house, then turned left and walked 20 km. He then turned East and walked 25 km and finally turning left covered 20 km. How far was he from my house?

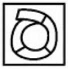
(a) 5 km (b) 10 km (c) 40 km (d) 80 km

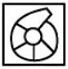
**103.** Find the mirror image in the given figure.



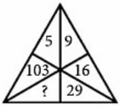
(a) 

(b) 

(c) 

(d) 

**104.** Find the missing character from the given alternatives.



(a) 54 (b) 55 (c) 56 (d) 57

**105.** The son of M is the father of N and grandfather (Mother's father) of R. S is the daughter of N and sister of B. On the basis of this information, how is M related to B?

(a) Grandfather (b) Grand mother

(c) Grandmother's mother

(d) Data inadequate

## SECTION-IV (MATHEMATICS)

**106.** If $a$, $b$, $c$ are roots of $y^3 - 3y^2 + 3y + 26 = 0$ and $\omega$ is cube root of unity, then the value of $\frac{a-1}{b-1} + \frac{b-1}{c-1} + \frac{c-1}{a-1}$ equals

(a) $-3\omega^2$ (b) $3\omega^2$ (c) $\omega^2$ (d) $2\omega^2$

**107.** If $z_n = \cos\left(\frac{n\pi}{10}\right) + i\sin\left(\frac{n\pi}{10}\right)$, then $\prod_{n=1}^{4} z_n$ equals

(a) 0 (b) 1

(c) $-1$ (d) none of these

**108.** The value of $x$ satisfying the equation

$|x-1|^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$ is

(a) 27 (b) 81 (c) 9 (d) 3

**109.** The roots of the equation $|x^2 - x - 6| = x + 2$ are

(a) $-2, 2, 4$ (b) $0, 1, 4$ (c) $0, 2, 4$ (d) $-2, 1, 4$

**110.** If $\sum_{r=0}^{\infty} a_r x^r = e^x(1+e^{4x})e^{-3x}$, then

$\sum_{p=1}^{\infty}(3^{2p-1}a_{2p-1}) =$

(a) $e^x$ (b) $e^2 - e^{-2}$

(c) 0 (d) none of these

**111.** 28 is divided into 4 parts which are in A.P. The ratio of the product of the first and third to the product of the second and fourth is 8 : 15. The largest part is

(a) 6 (b) 8 (c) 10 (d) 12

**112.** The number of ways you can find to pack 9 different books into five parcels if four of the parcels must contain two books each, is

(a) 945 (b) ${}^9C_2 \times {}^7C_2 \times {}^5C_2 \times {}^3C_2$

(c) ${}^9C_2 \times {}^7C_2 \times {}^5C_2 \times {}^3C_2 \times 5!$

(d) none of these

**113.** The number of permutations of letters A, B, C, D, E, F, G so that neither the pattern BEG nor CAD appears is

(a) $\dfrac{7!}{3!3!}$ (b) $\dfrac{7!}{2!3!3!}$
(c) 4806 (d) none of these

**114.** Numerically the greatest term in the expansion of $(2+3x)^{12}$, when $x=\dfrac{5}{6}$ is

(a) ${}^{12}C_7\, 2^4\left(\dfrac{5}{2}\right)^7$ (b) ${}^{12}C_5\, 2^5\left(\dfrac{5}{2}\right)^7$
(c) ${}^{12}C_5\, 2^5\left(\dfrac{5}{4}\right)^7$ (d) ${}^{12}C_5\, 2^3\left(\dfrac{5}{2}\right)^5$

**115.** Let $t$ be a positive integer and

$$\Delta_t = \begin{vmatrix} 2t-1 & m^2-1 & \cos^2(m^2) \\ {}^mC_t & 2^m & \cos^2(m) \\ 1 & m+1 & \cos(m^2) \end{vmatrix},$$ then the value of

$\sum\limits_{t=0}^{m} \Delta_t$ is equal to

(a) $2^m$ (b) 0
(c) $2^m\cos^2(2^m)$ (d) $m^2$

**116.** The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(a) no real roots (b) exactly one real root
(c) exactly four real roots (d) infinite real roots

**117.** If the expansion in powers of $x$ of the function $\dfrac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$, then $a_n$ is

(a) $\dfrac{b^n-a^n}{b-a}$ (b) $\dfrac{a^n-b^n}{b-a}$
(c) $\dfrac{a^{n+1}-b^{n+1}}{b-a}$ (d) $\dfrac{b^{n+1}-a^{n+1}}{b-a}$

**118.** If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$,

then $f(100)$ equals

(a) 0 (b) 1 (c) 100 (d) −100

**119.** If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then $x^2+y^2$ is

(a) 2 (b) 1 (c) 3 (d) 4

**120.** If a circle passes through the point $(a, b)$ and cuts the circle $x^2+y^2=k^2$ orthogonally, then the equation of the locus of its centre is

(a) $2ax + 2by - (a^2+b^2+k^2) = 0$
(b) $2ax + 2by - (a^2-b^2+k^2) = 0$
(c) $x^2+y^2-3ax-4by+(a^2+b^2-k^2)=0$
(d) $x^2+y^2-2ax-3by+(a^2+b^2-k^2)=0$

**121.** The equation of the plane through the line of intersection of planes $ax+by+cz+d=0$, $a'x+b'y+c'z+d'=0$ and parallel to the lines $y=0=z$ is

(a) $(ab'-a'b)x+(bc'-b'c)y+(ad'-a'd)=0$
(b) $(ab'-a'b)y+(ac'-a'c)z+(ad'-a'd)=0$
(c) $(ab'-a'b)x+(bc'-b'c)z+(ad'-a'd)=0$
(d) none of these

**122.** Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where $p$ is a constant.

Then $\dfrac{d^3}{dx^3}f(x)$ at $x=0$ is

(a) $p$ (b) $p+p^2$
(c) $p+p^3$ (d) independent of $p$

**123.** $\displaystyle\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt =$

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{2}$ (d) $\pi$

**124.** If $y^2 = P(x)$, a polynomial of degree 3, then $2\dfrac{d}{dx}\left(y^3\dfrac{d^2y}{dx^2}\right)$ equals

(a) $P'''(x)+P'(x)$ (b) $P''(x)\,P'''(x)$
(c) $P(x)\,P'''(x)$ (d) a constant

**125.** The probability that a teacher will given a unannounced test is $\dfrac{1}{4}$. If a student was absent twice, then probability that he will missed at least one test is

(a) $\dfrac{1}{2}$ (b) $\dfrac{7}{16}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{3}{4}$

**126.** If a unit vector $\vec{c}$ makes an angle $\dfrac{\pi}{6}$ with $\hat{i}\times\hat{j}$, then the maximum value of $(\hat{i}\times\hat{j})\cdot\vec{c}$ is

(a) 0 (b) $\dfrac{\sqrt{3}}{2}$
(c) 1 (d) none of these

**127.** If $x_1, x_2, \ldots, x_{18}$ are observations such that $\sum\limits_{j=1}^{18}(x_j-8)=9$ and $\sum\limits_{j=1}^{18}(x_j-8)^2=45$, then the standard deviation of these observations is

(a) $\sqrt{\dfrac{81}{34}}$ (b) 5 (c) $\sqrt{5}$ (d) $\dfrac{3}{2}$

**128.** The maximum value of $Z = 3x + 2y$, subjected to $x + 2y \leq 2$, $x + 2y \geq 8$; $x, y \geq 0$ is

(a) 32 (b) 24
(c) 40 (d) none of these

**129.** Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then

(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$
(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

**130.** The focal chord to $y^2 = 16x$ is tangent to $(x - 6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are

(a) $\{-1, 1\}$ (b) $\{-2, 2\}$ (c) $\{-2, 1/2\}$ (d) $\{2, -1/2\}$

**131.** The position vector of the point where the line $\vec{r} = \hat{i} - \hat{j} + \hat{k} + t(\hat{i} + \hat{j} - \hat{k})$ meets the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 5$ is

(a) $5\hat{i} + \hat{j} - \hat{k}$ (b) $5\hat{i} + 3\hat{j} - 3\hat{k}$
(c) $2\hat{i} + \hat{j} + 2\hat{k}$ (d) $5\hat{i} + \hat{j} + \hat{k}$

**132.** The value of $f(0)$ so that the function $f(x) = \frac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$ becomes continuous, is equal to

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) 2 (d) $\frac{1}{3}$

**133.** The differential equation of all 'Simple Harmonic Motions' of given period $\frac{2\pi}{n}$, where $x = a\cos(nt + b)$ is

(a) $\frac{d^2x}{dt^2} + nx = 0$ (b) $\frac{d^2x}{dt^2} + n^2x = 0$
(c) $\frac{d^2x}{dt^2} - n^2x = 0$ (d) $\frac{d^2x}{dt^2} + \frac{1}{n^2}x = 0$

**134.** The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left[[x] + \ln\left(\frac{1+x}{1-x}\right)\right] dx$ equals

(a) $\frac{-1}{2}$ (b) 0
(c) 1 (d) $2\ln\left(\frac{1}{2}\right)$

**135.** The statement $\sim (p \leftrightarrow \sim q)$ is

(a) equivalent to $\sim p \leftrightarrow q$
(b) a tautology
(c) a fallacy (d) equivalent to $p \leftrightarrow q$

**136.** If $q$ is the angle between the lines $AB$ and $AC$ where $A$, $B$ and $C$ are the three points with coordinates $(1, 2, -1)$, $(2, 0, 3)$, $(3, -1, 2)$ respectively, then $\sqrt{462}\cos\theta$ is equal to

(a) 20 (b) 10 (c) 30 (d) 40

**137.** The area of the smaller portion enclosed by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$, (in sq. units) is

(a) $\frac{\sqrt{2}}{3} + \frac{9\pi}{4} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)$

(b) $2\left(\frac{\sqrt{2}}{3} + \frac{9\pi}{4} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right)$

(c) $2\left(\frac{\sqrt{2}}{3} + \frac{9\pi}{4} + \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right)$

(d) $\frac{\sqrt{2}}{3} + \frac{9\pi}{4} + \frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)$

**138.** If $a$ and $b$ are positive numbers such that $a > b$, then the minimum value of $a\sec\theta - b\tan\theta$ $\left(0 < \theta < \frac{\pi}{2}\right)$, is

(a) $\frac{1}{\sqrt{a^2 - b^2}}$ (b) $\frac{1}{\sqrt{a^2 + b^2}}$
(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

**139.** The shortest distance between the parabolas $y^2 = 4x$ and $y^2 - 2x + 6 = 0$ is

(a) $\sqrt{2}$ units (b) $\sqrt{5}$ units
(c) $\sqrt{3}$ units (d) none of these

**140.** The mean of five observations is 4 and their variance is 5.2. If three of these observations are 2, 4 and 6, then the other two observations are

(a) 3 and 5 (b) 2 and 6 (c) 4 and 4 (d) 1 and 7

**141.** If $\int \frac{\cos 8x + 1}{\tan 2x - \cot 2x} dx = a\cos 8x + c$, then $a =$

(a) $-\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $-\frac{1}{8}$

**142.** If $\tan^{-1}(x - 1) + \tan^{-1}x + \tan^{-1}(x + 1) = \tan^{-1}3x$, then $x$ is

(a) $\pm\frac{1}{2}$ (b) $0, \frac{1}{2}$ (c) $0, -\frac{1}{2}$ (d) $0, \pm\frac{1}{2}$

**143.** The ellipse $E_1: \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle $R$ whose sides are parallel to the coordinate axes. Another ellipse $E_2$ passing through the point $(0, 4)$ circumscribes the rectangle $R$. The eccentricity of the ellipse $E_2$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

**144.** The shortest distance between the skew lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 3\hat{j} + 2\hat{k})$ and $\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$ is

(a) 3 units (b) $2\sqrt{3}$ units
(c) $\sqrt{3}$ units (d) $\sqrt{6}$ units

**145.** Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (in sq. units) is

(a) $ab$ (b) $2ab$ (c) $a/b$ (d) $\sqrt{ab}$

**146.** The solution of the differential equation

$$\frac{x\frac{dy}{dx}-y}{\sqrt{x^2-y^2}}=10x^2 \text{ is}$$

(a) $\sin^{-1}\left(\frac{y}{x}\right)-5x^2=C$ (b) $\sin^{-1}\left(\frac{y}{x}\right)=10x^2+C$

(c) $\frac{y}{x}=5x^2+C$

(d) $\sin^{-1}\left(\frac{y}{x}\right)=10x^2+Cx$

**147.** $E$ and $F$ are two independent events. The probability that both $E$ and $F$ happen is 1/12 and the probability that neither $E$ nor $F$ happens is 1/2, then

(a) $P(E) = 1/3, P(F) = 1/4$

(b) $P(E) = 1/2, P(F) = 1/6$

(c) $P(E) = 1/6, P(F) = 1/2$

(d) $P(E) = 1/2, P(F) = 1/3$

**148.** In a survey, it is found that 70% of employees like bananas and 64% like apples. If $x$ % like both bananas and apples, then

(a) $x \geq 34$ (b) $x \leq 64$

(c) $34 \leq x \leq 64$ (d) all of these

**149.** Let $\vec{a}=2\hat{i}+\hat{j}+\hat{k},\ \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ =

(a) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{3}}(-\hat{i}-\hat{j}-\hat{k})$

(c) $\frac{1}{\sqrt{5}}(-\hat{i}-2\hat{j})$ (d) $\frac{1}{\sqrt{3}}(-\hat{j}-\hat{k})$

**150.** If $y = e^x \cdot e^{x^2} \cdot e^{x^3} \ldots. e^{x^n} \ldots.$, for $0 < x < 1$, then $\frac{dy}{dx}$ at $x=\frac{1}{2}$ is

(a) $e$ (b) $4e$ (c) $2e$ (d) $3e$

## SOLUTIONS

**1. (d)**

**2. (b) :** Here, $T_{1/2}$ = 5.25 years, $t$ = 21 years

No. of half lives, $n=\frac{t}{T_{1/2}}=\frac{21 \text{ years}}{5.25 \text{ years}}=4$

Amount of sample left

$$=\left(\frac{1}{2}\right)^4 \times 1\text{ g}=\frac{1}{16}\text{ g}=\frac{1000}{16}\text{ mg}=62.5\text{ mg}$$

**3. (b) :** At constant temperature, $pV$ = constant

$$p_1V_1 = p_2V_2 \quad \text{or} \quad \frac{p_1}{p_2}=\frac{V_2}{V_1}$$

Fractional change in volume

$$\frac{V_1-V_2}{V_1}=\frac{4}{100}=\frac{1}{25} \Rightarrow \frac{V_2}{V_1}=\frac{24}{25}$$

$$\therefore \quad \frac{p_1}{p_2}=\frac{V_2}{V_1}=\frac{24}{25} \quad \text{or} \quad \frac{p_2}{p_1}=\frac{25}{24} \Rightarrow \frac{p_2-p_1}{p_1}=\frac{1}{24}$$

% increase in pressure = 100/24 = 4.17 %

**4. (a)**

**5. (a) :** Here, Input resistance, $R_i$ = 100 Ω

Change in base current, $\Delta I_B$ = 40 μA

Change in collector current, $\Delta I_C$ = 2 mA

Load resistance, $R_L$ = 4 kΩ = 4 × $10^3$ Ω

Current gain, $\beta=\frac{\Delta I_C}{\Delta I_B}=\frac{2\text{ mA}}{40\,\mu\text{A}}=\frac{2\times10^{-3}\text{ A}}{40\times10^{-6}\text{ A}}=50$

Voltage gain of the amplifier is

$$A_V=\beta\frac{R_L}{R_i}=50\times\frac{4\times10^3}{100}=2000$$

**6. (a)** **7. (c)**

**8. (c) :** $x = A\sin(\omega t + \pi/2) = A\cos\omega t$

$\therefore \quad \cos\omega t = x/A$ and $\sin\omega t=\sqrt{1-(x^2/A^2)}$

$y = A\sin 2\omega t = 2A\sin\omega t\cos\omega t$

or $y^2 = 4A^2\sin^2\omega t\cos^2\omega t$

$$=4A^2\times\frac{x^2}{A^2}\times\left(\frac{A^2-x^2}{A^2}\right)=4x^2\left(1-\frac{x^2}{A^2}\right)$$

**9. (d) :** 1 calorie = 4.2 J $\therefore$ [calorie] = $[\text{ML}^2\text{T}^{-2}]$.

Comparing with general dimensional formula $[\text{M}^a\text{L}^b\text{T}^c]$, we get $a$ = 1, $b$ = 2, $c$ = –2

$$\because \quad n_2=n_1\left[\frac{\text{M}_1}{\text{M}_2}\right]^a\left[\frac{\text{L}_1}{\text{L}_2}\right]^b\left[\frac{\text{T}_1}{\text{T}_2}\right]^c$$

$$\Rightarrow \quad n_2=4.2\left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1\left[\frac{1\text{ m}}{\beta\text{ m}}\right]^2\left[\frac{1\text{ s}}{\gamma\text{ s}}\right]^{-2}=4.2\,\alpha^{-1}\beta^{-2}\gamma^2$$

**10. (a) :** The hydrogen atom in the ground state (–13.6 eV) must be excited to the energy level with $n$ = 4 (*i.e.*, – 0.85 eV) so as to have $H_\beta$ line.

**11. (b)** **12. (a)**

**13. (b) :** $\beta=\frac{\Delta I_C}{\Delta I_B}=\frac{\Delta I_C}{\Delta I_E-\Delta I_C}$ $(\because \Delta I_E=\Delta I_B+\Delta I_C)$

or $100=\frac{1}{\Delta I_E-1}$ or $\Delta I_E-1=\frac{1}{100}=0.01\text{ mA}$

or $\Delta I_E$ = (1 + 0.01) mA = 1.01 mA

**14. (b)**

**15. (c) :** Work function, $\phi_0=\frac{hc}{\lambda_{\text{longest}}}$

$$\text{or} \quad \lambda_{\text{longest}}=\frac{hc}{\phi_0}=\frac{(6.63\times10^{-34})\times(3\times10^8)}{4.0\times1.6\times10^{-19}}$$

$= 310 \times 10^{-9}$ m = 310 nm

**16. (b)**

**17. (c) :** For sonometer, frequency of vibration of string, provided its tension and mass per unit length remain same, is given by $\upsilon\propto\frac{1}{l}$

$$\therefore\quad \frac{\upsilon_2}{\upsilon_1}=\frac{l_1}{l_2}\quad\text{or}\quad \frac{\upsilon_2}{256}=\frac{25}{16}\ \Rightarrow\ \upsilon_2=\frac{256\times 25}{16}=400\ \text{Hz}$$

**18. (a)** **19. (c)** **20. (b)**

**21. (b) :** According to equation of continuity, $a_1v_1 = a_2v_2$

$$\therefore\quad (40\times10^{-8})\times v_1 = 8\times10^{-4}\times\left(\frac{0.15}{60}\right)$$

$$\text{or}\quad v_1=\frac{8\times10^{-4}\times0.15}{40\times10^{-8}\times60}=5\ \text{m s}^{-1}$$

**22. (b) :** Let $h$ be maximum height reached by the body. Taking motion of the body from half the maximum height upto the highest point, we have

$u = 10$ m s$^{-1}$, $a = -g = -10$ m s$^{-2}$, $v = 0$, $S = h/2$

As $v^2 = u^2 + 2aS$

$\therefore\quad 0 = 10^2 + 2(-10)\times h/2$ or $h = 10$ m

**23. (c) :** According to Kepler's third law $T^2 \propto R^3$

$$T_2=T_1\left(\frac{R_2}{R_1}\right)^{3/2}=4\left(\frac{3R}{R}\right)^{3/2}=4\sqrt{27}\ \text{hours}$$

**24. (c)** **25. (a)** **26. (c)**

**27. (c) :** Potential difference across wire $= I\times 3=\dfrac{2\times3}{(3+R)}$

$$\text{Potential gradient}=\frac{6}{(3+R)}\frac{1}{100}=10^{-3}\ \text{V cm}^{-1}\qquad\text{(Given)}$$

or $\quad 60 = 3 + R$ or $R = 57\ \Omega$

**28. (b) :** Here, $d = 1$ mm $= 10^{-3}$ m, $\lambda = 6.5\times10^{-7}$ m, $D = 1$ m

$$x_5=n\lambda\frac{D}{d}=5\times6.5\times10^{-7}\times\frac{1}{10^{-3}}=32.5\times10^{-4}\ \text{m}$$

$$x_3=(2n-1)\frac{\lambda}{2}\frac{D}{d}=\frac{(2\times3-1)\times6.5\times10^{-7}\times1}{2\times10^{-3}}=16.25\times10^{-4}\ \text{m}$$

$$x_5-x_3=(32.5-16.25)\times10^{-4}\ \text{m}=16.25\times10^{-4}\ \text{m}=1.63\ \text{mm}$$

**29. (c):** Due to change in temperature $t$°C, increase in length,

$$\Delta l = l\alpha t\quad\text{or}\quad \frac{\Delta l}{l}=\alpha t$$

$$Y=\frac{\text{stress}}{\text{strain}}=\frac{\text{stress}}{\Delta l/l}\quad\therefore\quad \text{Stress}=Y\times\frac{\Delta l}{l}=Y\alpha t$$

**30. (d) :** Resistance of coil, $R=\dfrac{200\ \text{V}}{1\ \text{A}}=200\ \Omega$

With ac source, $I=\dfrac{200}{\sqrt{R^2+X_L^2}}$ or $0.5=\dfrac{200}{\sqrt{R^2+X_L^2}}$

or $\quad R^2 + (2\pi\upsilon L)^2 = (400)^2$

$$\text{or}\quad \left(2\pi\upsilon\times\frac{2\sqrt{3}}{\pi}\right)^2=(400)^2-(200)^2=200\times600$$

or $\quad 4\sqrt{3}\upsilon = 2\sqrt{3}\times100$ or $\upsilon = 50$ Hz.

**31. (a)**

**32. (b) :** $\dfrac{3h}{2\pi}=n\left(\dfrac{h}{2\pi}\right)\quad\therefore\quad n=3$

The kinetic energy of the electron in $n^{\text{th}}$ orbit is

$$K_n=\frac{13.6}{n^2}\ \text{eV}\quad\therefore\quad K_3=\frac{13.6}{3^2}\ \text{eV}=\frac{13.6}{9}\ \text{eV}=1.51\ \text{eV}$$

**33. (a)** **34. (a)**

**35. (c) :** Mutual inductance of the system, $M = \mu_0 n_1 n_2 A_2 l$ where $A_2$ is the area of inner solenoid.

$n_1 = 10\ \text{cm}^{-1} = 1000\ \text{m}^{-1}$, $n_2 = 40\ \text{cm}^{-1} = 4000\ \text{m}^{-1}$

$l = 30$ cm $= 30\times10^{-2}$ m, $A_2 = 20\ \text{cm}^2 = 20\times10^{-4}\ \text{m}^2$

$$\therefore\quad M = 4\pi\times10^{-7}\times1000\times4000\times20\times10^{-4}\times30\times10^{-2}$$
$$= 301.44\times10^{-5}\ \text{H} = 3\ \text{mH}$$

**36. (c)**

**37. (c) :** As $\mu=\dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

Here, $\delta_m = A$

$$\therefore\quad \mu=\frac{\sin A}{\sin\left(\frac{A}{2}\right)}=\frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}=2\cos\left(\frac{A}{2}\right)$$

$$\Rightarrow\quad A=2\cos^{-1}\left(\frac{\mu}{2}\right)$$

**38. (b)** **39. (b)** **40. (c)**

**41. (a) :** Acidity decreases as the electronegativity of the central halogen decreases from Cl to I in H$X$O (oxoacids).

**42. (b)**

**43. (c) :** Higher the reduction potential value of a species, greater is its tendency to undergo reduction and stronger is the oxidising power.

$$\begin{array}{lccc} & \text{BrO}_4^- & > \ \text{IO}_4^- & > \ \text{ClO}_4^- \\ E^\circ/\text{V} & 1.74 & 1.65 & 1.19 \end{array}$$

**44. (d) :** For $\text{HIn} \rightleftharpoons \text{H}^+ + \text{In}^-$

$$K_a=\frac{[\text{H}^+][\text{In}^-]}{[\text{HIn}]}$$

$$\therefore\quad \text{pH}=\text{p}K_a+\log\frac{[\text{In}^-]}{[\text{HIn}]}\quad\text{or}\quad \text{pH}-\text{p}K_a=\log\frac{[\text{In}^-]}{[\text{HIn}]}$$

**45. (b) :** 250 mg of $X$ is present in 100 mL of colloidal sol of gold. By definition, gold number of $X$ is that amount of it in mg which is present in 10 mL of colloidal gold solution. Hence, in 10 mL the amount of $X$ present is 25 mg, which is the gold number of $X$.

**46. (c) :** Pure hydrogen is evolved by reacting absolute alcohol and Na.

$C_2H_5OH + Na \rightarrow C_2H_5ONa + 1/2H_2$

Other statements are correct.

**47. (d)**

**48. (a) :** Electronic configuration of $\text{Zn}^{2+} = [\text{Ar}]3d^{10}4s^0$ ; $\text{Ni}^{2+} = [\text{Ar}]3d^84s^0$ ; $\text{Cr}^{3+} = [\text{Ar}]\ 3d^3\ 4s^0$. Hence, $\text{Zn}^{2+}$ is colourless (no unpaired electrons), $\text{Ni}^{2+}$ and $\text{Cr}^{3+}$ are coloured due to unpaired electrons.

**49. (b) :** Number of atoms present in the unit cell

$$Z=\frac{d\times a^3\times N_A}{M}$$

$$Z=\frac{(2.7\times10^3\,\text{kg m}^{-3})(4.05\times10^{-10}\,\text{m})^3\times(6.022\times10^{23}\,\text{mol}^{-1})}{(2.7\times10^{-2}\,\text{kg mol}^{-1})}=4$$

Since, there are four atoms per unit cell, the cubic unit cell must be face-centred.

**50. (b)** **51. (a)** **52. (b)**

**53. (d) :** ($O_2$ = 16 electrons)

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2, \pi 2p_y^2, \pi^* 2p_x^1, \pi^* 2p_y^1$

($N_2^+$ =13 electrons)

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x{}^2, \pi 2p_y{}^2, \sigma 2p_z{}^1$

Thus, it contains one unpaired electron, hence paramagnetic.

**54. (d)**

**55. (d) :** $E \propto \frac{1}{n^2}$ *i.e.*, when $n = 3$; $E$ decreases nine times.

**56. (b) :** Since the ratio of $p_A^\circ$ and $p_B^\circ$ is 1 : 2 and mole fraction is 1 : 2 therefore, partial pressure of $A$ ($p_A'$) $= p_A^\circ x_A$ and partial pressure of $B$ ($p_B'$) $= p_B^\circ x_B$ are related as

$p'_B = 4p'_A$

$P = p'_A + p'_B = p'_A + 4p'_A = 5p'_A$

The mole fraction of $A$ in the vapour in equilibrium with solution (according to Dalton's law of partial pressure) is

$$x'_A = \frac{p'_A}{P} = \frac{p'_A}{5p'_A} = \frac{1}{5} = 0.2$$

**57. (b)**

**58. (d) :** $B_2H_6 + 2N(CH_3)_3 \longrightarrow 2BH_3 \cdot N(CH_3)_3$

**59. (b) :** A mixture of 50% of *d* and 50% of *l* form is called racemic mixture.

**60. (b)** **61. (c)** **62. (a)**

**63. (b) :** $H_{2(g)} + CO_{2(g)} \rightleftharpoons H_2O_{(g)} + CO_{(g)}$

$\Delta G^\circ$ for reaction $= [G^\circ_{H_2O} + G^\circ_{CO}] - [G^\circ_{H_2} + G^\circ_{CO_2}]$

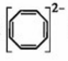
$= [-228.58 - 137.15] - [0 - 394.37] = 28.64$ kJ

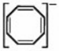
Also, $\Delta G = \Delta G^\circ + 2.303\ RT \log Q$

$= 28.64 + 2.303 \times 8.314 \times 10^{-3} \times 298 \log \frac{0.02 \times 0.01}{10 \times 20}$

$\Delta G = -5.61$ kJ

Thus, reaction will occur in forward direction.

**64. (d) :** Compound  has 8 + 2 = 10π-electrons, hence it is aromatic.

has 4π$e^-$ and has 8π$e^-$ then, these are antiaromatic compounds while  has 8 + 1 = 9π$e^-$, hence, it is non-aromatic.

**65. (b) :** $\overset{0}{I_2} \rightarrow 2\overset{-1}{I^-}$

$I_2$ undergoes reduction hence acts as oxidising agent.

$\overset{+2}{S_2}O_3^{2-} \rightarrow \overset{+2.5}{S_4}\ O_6^{2-}$

It undergoes oxidation hence acts as a reducing agent.

**66. (a)**

**67. (d) :** In this conformer Cl is at equatorial position and is least hindered.

**68. (c):** The higher the charge on the metal ion, smaller is the ionic size and more is the complex forming ability. Thus, the degree of complex formation decreases in the order :

$$M^{4+} > MO_2^{2+} > M^{3+} > MO_2^+$$

The higher tendency of complex formation of $MO_2^{2+}$ as compared to $M^{3+}$ is due to high concentration of charge on metal atom $M$ in $MO_2^{2+}$.

**69. (b) :** On increasing temperature, $K_w$ of water increases and hence p$K_w$ decreases.

**70. (a)**

**71. (d) :** Magnetic moment for an electron exists even if the orbital angular momentum is zero. This is explained by the spin magnetic moment because it is the spin which produces magnetic moment.

**72. (a)**

**73. (b) :** Amongst nitrobenzene, aniline, aniline hydrochloride and *N*-acetylaniline only aniline has electron donating $-NH_2$ group. As a result, electron density on the benzene ring increases and hence it becomes reactive towards electrophilic substitution reactions.

**74. (c):** Antihistamines are vascodialators and antiallergics.

**75. (a)**

**76. (c):** Activated charcoal is used for adsorbing poisonous gases.

**77. (c):** The *cis*-polyisoprene chains are held together by van der Waals forces and has a coiled structure. Thus, it can be stretched like a spring and exhibits elastic properties.

**78. (b)**

**79. (a) :** When $\Delta G_{sys} = 0$, the system is in equilibrium.

**80. (d) :** Proline has a 2° amino group. Hence, it is not estimated by this method.

**81. (b)** **82. (d)** **83. (a)** **84. (b)** **85. (d)**
**86. (a)** **87. (a)** **88. (b)** **89. (d)** **90. (c)**
**91. (d)** **92. (d)** **93. (a)** **94. (c)** **95. (c)**
**96. (b)** **97. (a)** **98. (d)** **99. (c)** **100. (b)**
**101. (d)** **102. (b)** **103. (c)** **104. (a)** **105. (d)**

**106. (b) :** From given equation $(y - 1)^3 = -27$

$$\Rightarrow \left(\frac{y-1}{-3}\right)^3 = 1 \Rightarrow \frac{y-1}{-3} = (1)^{1/3} \therefore \frac{y-1}{-3} = 1, \omega, \omega^2$$

As $a$, $b$, $c$ are roots.

$\therefore\ a - 1 = -3,\ b - 1 = -3\omega,\ c - 1 = -3\omega^2$

$$\therefore\ \frac{a-1}{b-1} + \frac{b-1}{c-1} + \frac{c-1}{a-1} = \frac{1}{\omega} + \frac{1}{\omega} + \frac{\omega^2}{\omega^3} = \omega^2 + \omega^2 + \omega^2 = 3\omega^2$$

**107. (c) :** $z_1\ z_2\ z_3\ z_4 = \prod_{n=1}^{4} z_n$

$$\therefore\ z_1z_2z_3z_4 = \left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)\left(\cos\frac{2\pi}{10} + i\sin\frac{2\pi}{10}\right) \times \left(\cos\frac{3\pi}{10} + i\sin\frac{3\pi}{10}\right)\left(\cos\frac{4\pi}{10} + i\sin\frac{4\pi}{10}\right)$$

$$= \cos\left(\frac{\pi}{10} + \frac{2\pi}{10} + \frac{3\pi}{10} + \frac{4\pi}{10}\right) + i\sin\left(\frac{\pi}{10} + \frac{2\pi}{10} + \frac{3\pi}{10} + \frac{4\pi}{10}\right)$$

$= \cos\pi + i\sin\pi = -1$

**108. (b)**

**109. (a):** $|x^2 - x - 6| = x + 2 \Rightarrow |(x + 2)(x - 3)| = x + 2$

$$\text{Now, } x + 2 = \begin{cases} (x+2)(x-3) & \forall\quad x \le -2 \text{ and } x \ge 3 \\ -(x^2 - x - 6) & \forall\quad -2 < x < 3 \end{cases}$$

If $(x + 2) = (x + 2)(x - 3)$, then $x = -2$, $x = 4$

and $(x + 2) = -(x + 2)(x - 3)$, then $x = -2$, $x = 2$

$\therefore\ x = -2, 2, 4$

**110. (c) :** $a_0 + a_1x + a_2x^2 + \ldots + \text{to } \infty = e^{2x} + e^{-2x}$

$$= 2\left\{1+\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}+\ldots\text{to }\infty\right\}$$

$\therefore\ a_1 = a_3 = a_5 = \ldots = 0$

$\therefore\ \sum_{p=1}^{\infty} 3^{2p-1}\ a_{2p-1} = 3{\cdot}a_1 + 3^3{\cdot}a_3 + 3^5{\cdot}a_5 + \ldots = 0$

**111. (c)**

**112. (a) :** Since 9 things are to be divided into 5 groups out of which four groups each have equal number 2 each and one group has only one element

Required number of ways $= \dfrac{9!}{2!2!2!2!4!} = 945.$

**113. (c) :** Total number of permutations = 7!

Let $A$ be the property that 'BEG' occurs.

$B$ be the property that 'CAD' occurs.

Number of permutations with $A = 5!$ = That of with $B$

And $(A \cap B) = 3!$

$\therefore\ n(A \cup B) = 5! + 5! - 3! = 234$

$\therefore$ Required number = 7! – 234 = 4806.

**114. (b)** **115. (b)**

**116. (a) :** Given, $e^{\sin x} - e^{-\sin x} - 4 = 0$

Let $e^{\sin x} = y$ $[y > 0\ \forall\ x \in R]$

then, $y - 1/y - 4 = 0$

$\Rightarrow\ y^2 - 4y - 1 = 0\ \Rightarrow\ y = \dfrac{4-\sqrt{16+4}}{2} = 2-\sqrt{5}$

$y$ can never be negative, so $2-\sqrt{5}$ can not be accepted.

Now, $2+\sqrt{5} > e$ and maximum value of $e^{\sin x} = e$

Hence, $e^{\sin x} \neq 2+\sqrt{5}$ *i.e.*, there is no solution (no real roots).

**117. (d)**

**118. (a) :** Taking $x$ common from $R_2$ and $x(x-1)$ from $R_3$, we get

$$f(x) = x^2(x-1)\begin{vmatrix} 1 & x & x+1 \\ 2 & x-1 & x+1 \\ 3 & x-2 & x+1 \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$, we get

$$f(x) = x^2(x-1)\begin{vmatrix} 1 & x+1 & x+1 \\ 2 & x+1 & x+1 \\ 3 & x+1 & x+1 \end{vmatrix} = x^2\ (x-1)\ (0) = 0$$

$\Rightarrow\ f(100) = 0.$

**119. (b) :** $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ ....(i)

and $x\sin\theta = y\cos\theta\ \ \therefore\ y = x\dfrac{\sin\theta}{\cos\theta}$ ....(ii)

Putting this value of $y$ in (i), we get

$x\sin^3\theta + x\dfrac{\sin\theta}{\cos\theta}\cdot\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow\ x\sin^3\theta\cos\theta + x\sin\theta\cos^3\theta = \sin\theta\cos^2\theta$

$\Rightarrow\ x = \dfrac{\cos\theta}{\sin^2\theta+\cos^2\theta} = \cos\theta$

Putting value of $x$ in (ii), we get

$y = \sin\theta\ \therefore\ x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$

**120. (a)** **121. (b)** **122. (d)**

**123. (a) :** Let $t = \sin^2 u$ in the first integral and $t = \cos^2 u$ in the second integral.

$$I_1 + I_2 = \int_0^x u\sin 2u\,du + \int_x^{\frac{\pi}{2}} u\sin 2u\,du = \int_0^{\frac{\pi}{2}} u\sin 2u\,du$$

$$= \left[-\frac{u}{2}\cos 2u + \frac{1}{4}\sin 2u\right]_0^{\pi/2} = \frac{\pi}{4}.$$

**124.(c)**

**125.(b) :** The probability that only one test was held in those days in which the student was absent

$$= \frac{1}{4}\times\frac{3}{4}+\frac{3}{4}\times\frac{1}{4} = 2\times\frac{1}{4}\times\frac{3}{4} = \frac{3}{8}$$

Now the probability that the test was held on both the days

$= \dfrac{1}{4}\times\dfrac{1}{4} = \dfrac{1}{16}$

$\therefore$ The probability that student missed at least one test

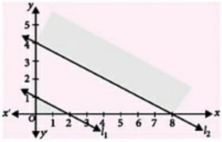
$= \dfrac{3}{8}+\dfrac{1}{16} = \dfrac{7}{16}$

**126.(b) :** $(\hat{i}\times\hat{j})\cdot\vec{c} \le |\hat{i}\times\hat{j}||\vec{c}|\cos\dfrac{\pi}{6} = |\hat{k}||\vec{c}|\cdot\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow\ (\hat{i}\times\hat{j})\cdot\vec{c} \le \dfrac{\sqrt{3}}{2}$

**127.(d)**

**128.(d) :** Let $l_1 : x + 2y = 2$, $l_2 : x + 2y = 8$,
$l_3 : x = 0$, $l_4 : y = 0$



Clearly, it has no solution because the constraints are inconsistent.

**129. (b) :** Since, $\theta \in \left(0, \dfrac{\pi}{4}\right) \Rightarrow \tan\theta < 1$ and $\cot\theta > 1$

Let $\tan\theta = 1 - x$ and $\cot\theta = 1 + y$ where $x, y > 0$ and are very small, then $t_1 = (1-x)^{1-x}$, $t_2 = (1-x)^{1+y}$

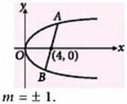
$t_3 = (1+y)^{1-x}$, $t_4 = (1+y)^{1+y}$

Clearly, $t_4 > t_3$ and $t_1 > t_2$. Also $t_3 > t_1$. Thus $t_4 > t_3 > t_1 > t_2$.

**130. (a) :** For parabola $y^2 = 16x$, focus = (4, 0)

Let $m$ be the slope of focal chord then equation is
$y = m\ (x-4)$ ...(i)

But according to question above is a tangent to the circle $(x-6)^2 + y^2 = 2$ which has $C(6, 0)$, $r = \sqrt{2}$



$\therefore$ Length of $\perp$ from (6, 0) to (i) = $r$

$\Rightarrow\ \dfrac{6m-4m}{\sqrt{m^2+1}} = \sqrt{2}$

$\Rightarrow\ 2m = \sqrt{2(m^2+1)}$

$\Rightarrow\ 2m^2 = m^2 + 1\ \Rightarrow\ m^2 = 1\ \Rightarrow\ m = \pm 1.$

**131. (b) :** Given equation of line is $\vec{r} = \hat{i}-\hat{j}+\hat{k}+t(\hat{i}+\hat{j}-\hat{k})$

$\Rightarrow\ \vec{r} = (1+t)\hat{i}+(t-1)\hat{j}+(1-t)\hat{k}$ ...(i)

Given, equation of plane is $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=5$

Using (i), we get $(1 + t) + (t - 1) + (1 - t) = 5 \Rightarrow t = 4$

Substituting this value in (i), we get $\vec{r}=5\hat{i}+3\hat{j}-3\hat{k}$

**132. (a)** **133. (b)**

**134. (a) :** Let $I = \int_{\frac{-1}{2}}^{\frac{1}{2}}\left[[x]+\ln\left(\frac{1+x}{1-x}\right)\right]dx$

Now, $\int_{\frac{-1}{2}}^{\frac{1}{2}}[x]dx=\int_{\frac{-1}{2}}^{0}[x]dx+\int_{0}^{\frac{1}{2}}[x]dx=\int_{\frac{-1}{2}}^{0}(-1)dx+0$

$=[-x]_{\frac{-1}{2}}^{0}=\frac{-1}{2}.$

And $\int_{\frac{-1}{2}}^{\frac{1}{2}}\ln\left(\frac{1+x}{1-x}\right)dx=0 \quad \left(\because \ln\left(\frac{1+x}{1-x}\right)\text{is an odd function}\right)$

$\therefore \quad I=\frac{-1}{2}+0=\frac{-1}{2}.$

**135. (d)**

**136. (a) :** Since, the coordinates of $A$, $B$ and $C$ are $(1, 2, -1)$, $(2, 0, 3)$ and $(3, -1, 2)$ respectively, then

$\overrightarrow{AB}=(2-1)\hat{i}+(0-2)\hat{j}+(3+1)\hat{k}=\hat{i}-2\hat{j}+4\hat{k}$

and $\overrightarrow{AC}=(3-1)\hat{i}+(-1-2)\hat{j}+(2+1)\hat{k}=2\hat{i}-3\hat{j}+3\hat{k}$

$\therefore \quad \cos\theta=\frac{(\hat{i}-2\hat{j}+4\hat{k})\cdot(2\hat{i}-3\hat{j}+3\hat{k})}{\sqrt{1+4+16}\sqrt{4+9+9}}=\frac{2+6+12}{\sqrt{21}\sqrt{22}}=\frac{20}{\sqrt{462}}$

$\Rightarrow \quad \sqrt{462}\cos\theta=20.$

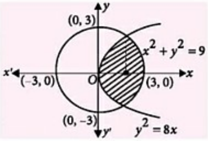
**137. (b) :** $x^2 + y^2 = 9$,

$x^2 + 8x - 9 = 0$

$\Rightarrow x=\frac{-8\pm\sqrt{64+36}}{2}$

$\Rightarrow x=\frac{-8\pm 10}{2}=-9,1$

$\Rightarrow x = 1$



Area enclosed $=2\left[\int_0^1 2\sqrt{2}\sqrt{x}dx+\int_1^3\sqrt{9-x^2}dx\right]$

$=2\left[2\sqrt{2}\int_0^1\sqrt{x}dx+\int_1^3\sqrt{9-x^2}dx\right]$

On simplifying, we get

Area $=2\left[\frac{\sqrt{2}}{3}+\frac{9\pi}{4}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}\right)\right]$ sq. units

**138. (d) :** Let $a\sec\theta - b\tan\theta = x \Rightarrow a^2\sec^2\theta = (x + b\tan\theta)^2$

$\Rightarrow \quad a^2(1+\tan^2\theta) = x^2 + 2bx\tan\theta + b^2\tan^2\theta$

$\Rightarrow \quad \tan^2\theta\,(a^2 - b^2) - 2bx\tan\theta + a^2 - x^2 = 0$

$\Rightarrow \quad \left(\tan\theta-\frac{bx}{a^2-b^2}\right)^2=\frac{a^2[x^2+b^2-a^2]}{(a^2-b^2)^2}$

Since L.H.S. is perfect square, so it is always +ve.

So, R.H.S. will be +ve.

When $x^2 + (b^2 - a^2) \geq 0 \Rightarrow x^2 \geq a^2 - b^2$

Thus, the minimum value of $x$ is $\sqrt{a^2-b^2}$ .

**139. (b)**

**140. (d):** Let the unknown data be $a$ and $b$.

Variance $=\frac{1}{n}\sum x^2-\bar{x}^2=5.2$

$\Rightarrow \quad \frac{1}{5}(4+16+36+a^2+b^2)-16=5.2$

$\Rightarrow \quad a^2 + b^2 = 50$ ...(i)

Also, $2 + 4 + 6 + a + b = 20 \Rightarrow a + b = 8$ ...(ii)

It is clear that the two observations are 1 and 7 which satisfies both (i) and (ii).

**141. (c) :** L.H.S. $=\int\frac{\cos 8x+1}{\tan 2x-\cot 2x}dx$

$=\int\frac{2\cos^2 4x}{\left(\frac{\sin^2 2x-\cos^2 2x}{\sin 2x\cos 2x}\right)}dx=-\int\frac{\cos^2 4x(2\sin 2x\cos 2x)}{(\cos^2 2x-\sin^2 2x)}dx$

$=-\int\frac{\cos^2 4x\times\sin 4x}{\cos 4x}dx=-\frac{1}{2}\int 2\sin 4x\cos 4x\,dx$

$=-\frac{1}{2}\int\sin 8x\,dx=\frac{1}{2}\times\frac{\cos 8x}{8}+c$

Now, $\frac{1}{2}\frac{\cos 8x}{8}+c=a\cos 8x+c \Rightarrow a=\frac{1}{16}$

**142. (d)**

**143. (c) :** Let the required ellipse be $\frac{x^2}{\alpha}+\frac{y^2}{\beta}=1$

It passes through (0, 4) $\Rightarrow \frac{16}{\beta}=1 \Rightarrow \beta=16$

It passes through (3, 2) $\Rightarrow \frac{9}{\alpha}+\frac{4}{\beta}=1 \Rightarrow \alpha=12$

Also $\alpha = \beta(1 - e^2) \Rightarrow 12 = 16(1 - e^2)$

$\Rightarrow \frac{3}{4}=1-e^2 \Rightarrow e^2=\frac{1}{4} \Rightarrow e=\frac{1}{2}$

**144. (c) :** Shortest distance $=\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&3&2\\2&3&1\end{vmatrix}=\hat{i}(-3)-\hat{j}(-3)+\hat{k}(-3)=-3\hat{i}+3\hat{j}-3\hat{k}$

$\therefore \quad |\vec{b}_1\times\vec{b}_2|=\sqrt{9+9+9}=\sqrt{27}=3\sqrt{3}$

$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=(3\hat{i}+3\hat{j}+3\hat{k})\cdot(-3\hat{i}+3\hat{j}-3\hat{k})$

$= -9 + 9 - 9 = -9$

$\therefore$ Shortest distance $=\frac{9}{3\sqrt{3}}=\sqrt{3}$ units

**145. (b)** **146. (a)** **147. (a)**

**148. (d)** **149. (a)** **150. (b)**

◆

# CBSE warm-up!

CLASS-XII

Exam on 1st June 2021

**Practice questions for CBSE Exams as per the reduced syllabus and marking scheme issued by CBSE for the academic session 2020-21.**

## Practice Paper 2021

Time Allowed : 3 hours
Maximum Marks : 80

### GENERAL INSTRUCTIONS

(1) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
(2) Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
(3) Both Part A and Part B have internal choices.

**Part-A :**

(1) It consists of two sections- I and II.
(2) Section I comprises of 16 very short answer type questions.
(3) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

**Part-B :**

(1) It consists of three sections- III, IV and V.
(2) Section III comprises of 10 questions of 2 marks each.
(3) Section IV comprises of 7 questions of 3 marks each.
(4) Section V comprises of 3 questions of 5 marks each.
(5) Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

## PART-A

### SECTION-I

**All questions are compulsory. In case of internal choices attempt any one.**

**1.** If the function $f(x)=\begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$, then find the value of $k$.

**OR**

If $y = \log_7(\log x)$, then find $\frac{dy}{dx}$.

**2.** Write the principal value branch of $\operatorname{cosec}^{-1}x$.

**OR**

Find the principal values of $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$.

**3.** The position vectors of points $A$ and $B$ are $\hat{i}+3\hat{j}-7\hat{k}$ and $5\hat{i}-2\hat{j}+4\hat{k}$ respectively, then find the direction cosine of $\overrightarrow{AB}$ along $Y$-axis.

**4.** Write the direction cosines of a line parallel to the line $\frac{3-x}{3}=\frac{y+2}{-2}=\frac{z+2}{6}$.

**5.** Evaluate : $\displaystyle\int \frac{dx}{5-8x-x^2}$

**OR**

Evaluate : $\displaystyle\int_{-\pi/4}^{\pi/4} |\sin x|\,dx$

**6.** If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of $\alpha$, $A$ is an identity matrix?

**OR**

If $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$, then find the matrix $PQ$.

**7.** If $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line, then find the value of $n$.

**8.** Find the number of equivalence relations on the set {1, 2, 3} containing (1, 3) and (3, 1).

**9.** Find the order of the differential equation whose general solution is given by $y = (A + B)\cos(x + C) + De^x$.

**10.** Find the vector equation of a plane which is at a distance of 6 units from the origin and which has $\hat{k}$ as the unit vector normal to it.

**OR**

Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ units from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$.

**11.** If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, then find the values of $x$ and $y$.

**12.** Find the equation of a plane with intercepts 2, 3 and 4 on the $X$, $Y$ and $Z$-axes respectively.

**13.** If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, then find the value of $x$.

**14.** Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

**15.** If $R = \{(x, y) : x + 2y = 8\}$ is a relation on $N$, then find the range of $R$.

**16.** If $\alpha, \beta, \gamma$ are the angles made by a line with the co-ordinate axes. Then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

## SECTION - II

**Both the case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 and 18. Each sub-part carries 1 mark.**

**17.** In a school, a football game is to be organised between students of class 11th and 12th. For which, a team from each class is chosen, say $T_1$ be the team of class 11th and $T_2$ be the team of class 12th. These teams have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probability of $T_1$ winning, drawing and losing a game against $T_2$ are $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively.

Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game.

Let $X$ and $Y$ denote the total points scored by team $T_1$ and $T_2$, respectively, after two games.

Based on the above information answer the following questions :

**(i)** $P(T_2$ winning a match against $T_1)$ is equal to

(a) 1/2 (b) 1/6
(c) 1/3 (d) none of these

**(ii)** $P(T_2$ drawing a match against $T_1)$ is equal to

(a) 1/2 (b) 1/3 (c) 1/6 (d) 2/3

**(iii)** $P(X > Y)$ is equal to

(a) 1/4 (b) 5/12 (c) 1/2 (d) 7/12

**(iv)** $P(X = Y)$ is equal to

(a) 11/36 (b) 1/3 (c) 13/36 (d) 1/2

**(v)** $P(X + Y = 12)$ is equal to

(a) 0 (b) 5/12 (c) 13/36 (d) 7/12

LATEST 2020-21 PRACTICE PAPERS Included

As per the CBSE **Revised Curriculum**
For the Academic Year **2020-21**

MtG

## CBSE CHAMPION Chapterwise -Topicwise Solved Papers

CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE PHYSICS
CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE CHEMISTRY
CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE MATHEMATICS
CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE BIOLOGY
CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE ENGLISH CORE
CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE ECONOMICS
CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE BUSINESS STUDIES
CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE ACCOUNTANCY
CBSE 10 CHAMPION CHAPTERWISE-TOPICWISE PHYSICAL EDUCATION
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**18.** In a street two lamp posts are 300 feet apart. The light intensity at a distance $d$ from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance $d$ from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source).

The combined light intensity is the sum of the two light intensities coming from both lamp posts.

Based on the above information, answer the following questions.

**(i)** If you are in between the lamp posts, at distance $x$ feet from the stronger light, then the formula for the combined light intensity coming from both lamp posts as function of $x$, is

(a) $\frac{1000}{x^2}+\frac{125}{x^2}$  (b) $\frac{1000}{(300-x^2)}+\frac{125}{x^2}$

(c) $\frac{1000}{x^2}+\frac{125}{(300-x)^2}$

(d) None of these

**(ii)** The maximum value of $x$ can not be

(a) 100  (b) 200

(c) 300  (d) None of these

**(iii)** The minimum value of $x$ can not be

(a) 0  (b) 100

(c) 200  (d) None of these

**(iv)** If $I(x)$ denote the combined light intensity, then $I(x)$ will be minimum when $x$ =

(a) 100  (b) 200  (c) 300  (d) 150

**(v)** The darkest spot between the two lights is

(a) at a distance of 100 feet from the weaker lamp post.

(b) at distance of 100 feet from the stronger lamp post.

(c) at a distance of 200 feet from the weaker lamp post.

(d) None of these

## PART-B

## SECTION-III

**All questions are compulsory. In case of internal choices attempt any one.**

**19.** Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on $R$.

**20.** If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$, then find $\vec{a}\times\vec{b}$ and $|\vec{a}\times\vec{b}|$.

**21.** If $A$ and $B$ are events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$, then find $P(\text{not } A \text{ and not } B)$.

**22.** Find the number of triplets $(x, y, z)$ satisfying the equation $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$.

**23.** Evaluate : $\int (e^{x\log a}+e^{a\log x}+e^{a\log a})\,dx$

**OR**

Evaluate : $\int_0^{\infty}\frac{dx}{(x^2+4)(x^2+9)}$

**24.** Find the solution of the differential equation $\frac{dy}{dx}=\cos(x+y)$.

**25.** Given that the events $A$ and $B$ are such that $P(A)=\frac{1}{2}$, $P(A\cup B)=\frac{3}{5}$ and $P(B) = p$. Find $p$ if $A$ and $B$ are (i) mutually exclusive (ii) independent.

**OR**

A bag contains 12 white pearls and 18 black pearls. Two pearls are drawn in succession without replacement. Find the probability that the first pearl is white and the second is black.

**26.** Find the values of $a$ and $b$ such that the function $f$ defined by
$$f(x)=\begin{cases}\frac{x-4}{|x-4|}+a, & \text{if } x<4\\ a+b, & \text{if } x=4\\ \frac{x-4}{|x-4|}+b, & \text{if } x>4\end{cases}$$
is continuous at $x = 4$.

**27.** Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

**28.** Find $(AB)^{-1}$, if $A=\begin{bmatrix}1 & 0\\ -4 & 2\end{bmatrix}$ and $B^{-1}=\begin{bmatrix}3 & 1\\ 5 & 2\end{bmatrix}$.

**OR**

Evaluate the determinant $\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$.

## SECTION-IV

**All questions are compulsory. In case of internal choices attempt any one.**

**29.** Show that the relation $R$ in the set of real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive, nor symmetric, nor transitive.

**30.** Show that the height of the closed cylinder of given surface area and maximum volume, is equal to the diameter of base.

**31.** If $y = e^x \sin x^3 + (\tan x)^x$, then find $\dfrac{dy}{dx}$.

**OR**

If $x = 3 \sin t - \sin 3t$, $y = 3 \cos t - \cos 3t$, then find $\dfrac{d^2y}{dx^2}$ at $t = \dfrac{\pi}{3}$.

**32.** Evaluate : $\displaystyle\int \frac{dx}{1 - \cos x - \sin x}$

**33.** Check whether the function

$$f(x) = \begin{cases} \dfrac{(3^{\sin x} - 1)^2}{x \cdot \log(1+x)}, & \text{for } x \ne 0 \\ 2\log 3, & \text{for } x = 0 \end{cases}$$ is continuous

or not.

**OR**

If $x = 2 \cos \theta - \cos 2\theta$ and $y = 2 \sin \theta - \sin 2\theta$, find $\dfrac{d^2 y}{dx^2}$ at $\theta = \dfrac{\pi}{2}$.

**34.** Solve : $(x + y)^2 \dfrac{dy}{dx} = a^2$

**35.** Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

## SECTION-V

**All questions are compulsory. In case of internal choices attempt any one.**

**36.** The points $A(4, 5, 10)$, $B(2, 3, 4)$ and $C(1, 2, -1)$ are three vertices of a parallelogram $ABCD$. Find the vector and cartesian equations of the sides $AB$ and $BC$ and find coordinates of $D$.

**OR**

Find the equation of the plane passing through the point $(3, -3, 1)$ and perpendicular to the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$. Also find the coordinates of foot of perpendicular, the equation of perpendicular line and the length of perpendicular drawn from origin to the plane.

**37.** Find the number of points at which the objective function $z = 3x + 2y$ can be maximized subject to $3x + 5y \le 15$, $5x + 2y \le 20$, $x \ge 0$, $y \ge 0$.

**OR**

Maximise $Z = x + 2y$
subject to the constraints:
$x + 2y \ge 100$
$2x - y < 0$
$2x + y \le 200$
$x, y \ge 0$
Solve the above L.P.P. graphically.

**38.** Find the inverse of $A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$. Hence find $(A^{-1})^2$.

**OR**

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj $A$ and verify that $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_3$.

## SOLUTIONS

**1.** Since, $f(x)$ is continuous at $x = 2$.

$\therefore \displaystyle\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) \Rightarrow k(2)^2 = 3 \Rightarrow k = \frac{3}{4}$

**OR**

$y = \log_7 (\log x) = \dfrac{\log(\log x)}{\log 7}$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{\log 7} \cdot \dfrac{1}{\log x} \cdot \dfrac{1}{x} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\log 7 x \log x}$

**2.** Principal value branch of $\operatorname{cosec}^{-1} x$ is $\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right] - \{0\}$.

**OR**

Let $\cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right) = \theta \quad \Rightarrow \cos\theta = \dfrac{-\sqrt{3}}{2}$

$= -\cos\dfrac{\pi}{6} = \cos\left(\pi - \dfrac{\pi}{6}\right) = \cos\dfrac{5\pi}{6} \Rightarrow \theta = \dfrac{5\pi}{6} \in [0, \pi]$

$\therefore$ Principal value of $\cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$ is $\dfrac{5\pi}{6}$.

**3.** Here $\vec{a} = \hat{i} + 3\hat{j} - 7\hat{k}, \vec{b} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

$\overrightarrow{AB} = \vec{b} - \vec{a} = 4\hat{i} - 5\hat{j} + 11\hat{k}$

Direction cosine along $Y$-axis = $\dfrac{-5}{\sqrt{16+25+121}} = \dfrac{-5}{\sqrt{162}}$

**4.** We have, $\dfrac{x-3}{-3} = \dfrac{y+2}{-2} = \dfrac{z+2}{6}$

$\Rightarrow$ Direction ratios are $-3, -2, 6$.

$\therefore$ Direction cosines are $\dfrac{-3}{7}, \dfrac{-2}{7}, \dfrac{6}{7}$.

These are direction cosines of a line parallel to given line.

**5.** Let $I = \displaystyle\int \frac{dx}{5-8x-x^2} = \int \frac{dx}{21-(x+4)^2}$

$$= \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2} = \frac{1}{2\sqrt{21}} \log\left|\frac{\sqrt{21}+x+4}{\sqrt{21}-x-4}\right| + C$$

**OR**

Let $I = \displaystyle\int_{-\pi/4}^{\pi/4} |\sin x|\, dx = 2\int_{0}^{\pi/4} \sin x\, dx$

$$= 2\left[-\cos x\right]_0^{\pi/4} = -2\left[\frac{1}{\sqrt{2}} - 1\right] = 2 - \sqrt{2}$$

**6.** Given, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$. Now, $A$ is an identity matrix then, $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore$ $\cos\alpha = 1$ and $\sin\alpha = 0 \Rightarrow \alpha = 0^\circ$

**OR**

Since, $P = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$$\therefore \quad PQ = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix} \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2 - i^2 & i^2 + i^2 \\ i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix} = \begin{bmatrix} 1+1 & -1-1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$$

**7.** Since, $\left(\dfrac{1}{2}, \dfrac{1}{3}, n\right)$ are the direction cosines of a line.

$$\therefore \quad \left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + n^2 = 1 \Rightarrow n^2 = \frac{23}{36} \Rightarrow n = \frac{\pm\sqrt{23}}{6}$$

**8.** Equivalence relations on the set $\{1, 2, 3\}$ containing $(1, 3)$ and $(3, 1)$ are

$A_1 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$

$A_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 1), (3, 2), (1, 3)\}$

So, only two equivalence relations exist.

**9.** Given $y = (A + B) \cos (x + C) + De^x$

or $y = k \cos (x + C) + De^x$

Now order of a differential equation is same as the number of arbitrary unknowns present in the solution. Hence order of differential equation is 3.

**10.** Here, $\hat{n} = \hat{k}$ and $d = 6$

$\therefore$ $\vec{r} \cdot \hat{n} = d \Rightarrow \vec{r} \cdot \hat{k} = 6$, which is the required equation of plane.

**OR**

Let $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Then,

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{4+9+16}} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$$

Hence, the required equation of the plane is

$$\vec{r} \cdot \left(\frac{2}{\sqrt{29}}\hat{i} + \frac{(-3)}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}$$

or $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 6$

**11.** Given $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$

$\Rightarrow$ $y = -1$ and $7 - x = 0 \Rightarrow x = 7, y = -1$

**12.** As the plane has intercepts 2, 3 and 4 on $X$, $Y$ and $Z$ axes respectively.

$\therefore$ The required equation of the plane is

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1 \Rightarrow 6x + 4y + 3z = 12$$

**13.** Expanding the given determinant, we get

$x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta\,(-\sin\theta + x\cos\theta) = 8$

$\Rightarrow$ $-x^3 - x + x = 8 \Rightarrow x^3 + 8 = 0$

$\Rightarrow$ $(x + 2)(x^2 - 2x + 4) = 0$

$\Rightarrow$ $x + 2 = 0$ $\quad [\because x^2 - 2x + 4 > 0\ \forall x]$

$\Rightarrow$ $x = -2$

**14.** The given vectors are

$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$, $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

$\therefore$ Required sum $= \vec{a} + \vec{b} + \vec{c}$

$= (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) + (\hat{i} - 6\hat{j} - 7\hat{k}) = -4\hat{j} - \hat{k}$

**15.** Here, $R = \{(x, y) : x + 2y = 8\}$, where $x, y \in N$.
For $x = 1, 3, 5, \ldots ; x + 2y = 8$ has no solution in $N$.
For $x = 2$, we have $2 + 2y = 8 \Rightarrow y = 3$
For $x = 4$, we have $4 + 2y = 8 \Rightarrow y = 2$
For $x = 6$ , we have $6 + 2y = 8 \Rightarrow y = 1$
For $x = 8, 10, \ldots ; x + 2y = 8$ has no solution in $N$.
$\therefore$ Range of $R = \{1, 2, 3\}$

**16.** $\because \alpha, \beta$ and $\gamma$ are the angles made by line with the co-ordinate axes.

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$
$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

**17.** **(i) (c) :** Clearly, $P(\text{T}_2$ winning a match against $\text{T}_1)$ $= P(\text{T}_1$ losing$) = 1/3$

**(ii) (c) :** Clearly, $P(\text{T}_2$ drawing a match against $\text{T}_1)$ $= P(\text{T}_1$ drawing$) = 1/6$

**(iii) (b) :** According to given information, we have the following possibilities for the value of $X$ and $Y$.

| $X$ | 6 | 4 | 3 | 2 | 1 | 0 |
|---|---|---|---|---|---|---|
| $Y$ | 0 | 1 | 3 | 2 | 4 | 6 |

Now, $P(X > Y) = P(X = 6, Y = 0) + P(X = 4, Y = 1)$
$= P(\text{T}_1 \text{ win})\, P(\text{T}_1 \text{ win}) + P(\text{T}_1 \text{ win})\, P(\text{match draw})$
$+ P(\text{match draw})\, P(\text{T}_1 \text{ win})$

$$= \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{2} = \frac{3+1+1}{12} = \frac{5}{12}$$

**(iv) (c) :** $P(X = Y) = P(X = 3, Y = 3) + P(X = 2, Y = 2)$
$= P(\text{T}_1 \text{ win})\, P(\text{T}_2 \text{ win}) + P(\text{T}_2 \text{ win})\, P(\text{T}_1 \text{ win})$
$+ P(\text{match draw})\, P(\text{match draw})$

$$= \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{36} = \frac{13}{36}$$

**(v) (a) :** From the given information, it is clear that maximum sum of $X$ and $Y$ can be 6, therefore $P(X + Y = 12) = 0$

**18.** **(i) (c) :** Since, the distance is $x$ feet from the stronger light, therefore the distance from the weaker light will be $300 - x$.

So, the combined light intensity from both lamp posts is given by $\dfrac{1000}{x^2} + \dfrac{125}{(300-x)^2}$.

**(ii) (c) :** Since, the person is in between the lamp posts, therefore $x$ will lie in the interval $(0, 300)$.
So, maximum value of $x$ can't be 300.

**(iii) (a) :** Since, $0 < x < 300$, therefore minimum value of $x$ can't be 0.

**(iv) (b) :** We have, $I(x) = \dfrac{1000}{x^2} + \dfrac{125}{(300-x)^2}$

$$\Rightarrow I'(x) = \frac{-2000}{x^3} + \frac{250}{(300-x)^3} \text{ and}$$

$$\Rightarrow I''(x) = \frac{2000}{x^4} + \frac{750}{(300-x)^4}$$

For maxima/minima, $I'(x) = 0$

$$\Rightarrow \frac{2000}{x^3} = \frac{250}{(300-x)^3} \Rightarrow 8(300-x)^3 = x^3$$

Taking cube root on both sides, we get

$$2(300 - x) = x \Rightarrow 600 = 3x \Rightarrow x = 200$$

Thus, $I(x)$ is minimum when you are at 200 feet from the strong intensity lamp post.

**(v) (a) :** Since, $I(x)$ is minimum when $x = 200$ feet, therefore the darkest spot between the two light is at a distance of 200 feet from stronger lamp post, *i.e.*, at a distance of $300 - 200 = 100$ feet from the weaker lamp post.

**19.** We have, $f(x) = 4x^3 - 18x^2 + 27x - 7$
$\Rightarrow f'(x) = 12x^2 - 36x + 27$

$$= 12\left(x^2 - 3x + \frac{9}{4}\right) = 12\left(x - \frac{3}{2}\right)^2 \geq 0 \ \forall x \in R$$

Hence, $f(x)$ is always increasing on $R$.

## SAMURAI SUDOKU

### ANSWER - FEBRUARY 2021

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 9 | 8 | 7 | 5 | 3 | 4 | 6 | 1 | | | | 2 | 8 | 1 | 9 | 5 | 3 | 6 | 4 | 7 |
| 6 | 4 | 7 | 1 | 9 | 8 | 5 | 2 | 3 | | | | 5 | 7 | 3 | 2 | 4 | 6 | 1 | 8 | 9 |
| 3 | 1 | 5 | 2 | 4 | 6 | 9 | 8 | 7 | | | | 9 | 4 | 6 | 1 | 8 | 7 | 3 | 5 | 2 |
| 5 | 7 | 6 | 4 | 3 | 2 | 1 | 9 | 8 | | | | 8 | 2 | 5 | 4 | 3 | 9 | 7 | 1 | 6 |
| 9 | 8 | 1 | 5 | 6 | 7 | 2 | 3 | 4 | | | | 7 | 3 | 9 | 8 | 6 | 1 | 4 | 2 | 5 |
| 4 | 3 | 2 | 9 | 8 | 1 | 7 | 5 | 6 | | | | 1 | 6 | 4 | 5 | 7 | 2 | 8 | 9 | 3 |
| 8 | 5 | 9 | 6 | 7 | 4 | 3 | 1 | 2 | 5 | 6 | 8 | 4 | 9 | 7 | 6 | 1 | 5 | 2 | 3 | 8 |
| 1 | 6 | 4 | 3 | 2 | 9 | 8 | 7 | 5 | 3 | 9 | 4 | 6 | 1 | 2 | 3 | 9 | 8 | 5 | 7 | 4 |
| 7 | 2 | 3 | 8 | 1 | 5 | 6 | 4 | 9 | 1 | 2 | 7 | 3 | 5 | 8 | 7 | 2 | 4 | 9 | 6 | 1 |
| | | | | | | 9 | 3 | 8 | 2 | 5 | 1 | 7 | 4 | 6 | | | | | | |
| | | | | | | 1 | 2 | 4 | 7 | 3 | 6 | 5 | 8 | 9 | | | | | | |
| | | | | | | 7 | 5 | 6 | 8 | 4 | 9 | 1 | 2 | 3 | | | | | | |
| 2 | 1 | 6 | 8 | 7 | 4 | 5 | 9 | 3 | 6 | 1 | 2 | 8 | 7 | 4 | 3 | 1 | 5 | 2 | 9 | 6 |
| 4 | 9 | 8 | 3 | 5 | 1 | 2 | 6 | 7 | 4 | 8 | 5 | 9 | 3 | 1 | 6 | 2 | 8 | 7 | 5 | 4 |
| 3 | 7 | 5 | 6 | 2 | 9 | 4 | 8 | 1 | 9 | 7 | 3 | 2 | 6 | 5 | 7 | 4 | 9 | 3 | 8 | 1 |
| 1 | 4 | 3 | 2 | 9 | 6 | 8 | 7 | 5 | | | | 3 | 9 | 7 | 1 | 6 | 2 | 5 | 4 | 8 |
| 5 | 6 | 7 | 4 | 3 | 8 | 9 | 1 | 2 | | | | 5 | 1 | 2 | 8 | 7 | 4 | 9 | 6 | 3 |
| 8 | 2 | 9 | 5 | 1 | 7 | 3 | 4 | 6 | | | | 6 | 4 | 8 | 5 | 9 | 3 | 1 | 2 | 7 |
| 6 | 5 | 4 | 1 | 8 | 2 | 7 | 3 | 9 | | | | 1 | 2 | 9 | 4 | 3 | 6 | 8 | 7 | 5 |
| 7 | 8 | 2 | 9 | 6 | 3 | 1 | 5 | 4 | | | | 7 | 8 | 6 | 9 | 5 | 1 | 4 | 3 | 2 |
| 9 | 3 | 1 | 7 | 4 | 5 | 6 | 2 | 8 | | | | 4 | 5 | 3 | 2 | 8 | 7 | 6 | 1 | 9 |

**20.** Given, $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & 2 & 3 \end{vmatrix}$

$= (9+2)\hat{i} - (6+1)\hat{j} + (4-3)\hat{k} = 11\hat{i} - 7\hat{j} + \hat{k}$

$\therefore\ |\vec{a} \times \vec{b}| = \sqrt{(11)^2 + (-7)^2 + (1)^2} = \sqrt{171}$

**21.** Here, $P(A) \cdot P(B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12} = P(A \cap B)$

$\Rightarrow$ Events $A$ and $B$ are independent.

$\Rightarrow$ Events $\bar{A}$ and $\bar{B}$ are also independent.

Now, $P(\bar{A} \cap \bar{B}) = P(\bar{A})\ P(\bar{B})$

($\because$ $\bar{A}$ and $\bar{B}$ are independent events)

$= (1 - P(A))(1 - P(B))$

$= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

**22.** We have, $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

$\because\quad -\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}, \frac{-\pi}{2} \le \sin^{-1}y \le \frac{\pi}{2}$

and $\frac{-\pi}{2} \le \sin^{-1}z \le \frac{\pi}{2}$

$\therefore$ The above condition will true if

$\sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2} \Rightarrow x = y = z = 1$

Thus, there is only one triplet.

**23.** Let $I = \int (e^{x\log a} + e^{a\log x} + e^{a\log a})\,dx$

$= \int (e^{\log a^x} + e^{\log x^a} + e^{\log a^a})\,dx$

$= \int (a^x + x^a + a^a)\,dx \qquad [\because\ e^{\log\lambda} = \lambda]$

$= \int a^x dx + \int x^a dx + \int a^a dx = \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a x + c$

**OR**

Let $I = \int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$

$= \frac{1}{5}\left(\int_0^\infty \frac{1}{x^2+4}dx - \int_0^\infty \frac{1}{(x^2+9)}dx\right)$

$= \frac{1}{5}\left[\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^\infty - \left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]_0^\infty\right]$

$= \frac{1}{5}\left[\left(\frac{1}{2}\cdot\frac{\pi}{2} - 0\right) - \left(\frac{1}{3}\cdot\frac{\pi}{2} - 0\right)\right] = \frac{1}{5}\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\pi}{60}$

**24.** We have $\frac{dy}{dx} = \cos(x+y)$

Put $x + y = u \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$

$\therefore\quad \frac{du}{dx} - 1 = \cos u \Rightarrow \int \frac{du}{1+\cos u} = \int dx + c$

$\Rightarrow\quad \int \frac{1}{2}\sec^2\left(\frac{u}{2}\right)du = x + c \Rightarrow \tan\left(\frac{u}{2}\right) = x + c$

$\Rightarrow\quad \tan\left(\frac{x+y}{2}\right) = x + c$

**25.** **(i)** When $A$ and $B$ are mutually exclusive, then $A \cap B = \phi \Rightarrow P(A \cap B) = 0 \Rightarrow P(A \cup B) = P(A) + P(B)$

$\Rightarrow\quad \frac{3}{5} = \frac{1}{2} + p \ \Rightarrow\ p = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10} = \frac{1}{10}$

**(ii)** When $A$ and $B$ are independent, then

$P(A \cap B) = P(A)\ P(B)$

$\Rightarrow\quad P(A \cup B) = P(A) + P(B) - P(A)\ P(B)$

$\Rightarrow\quad \frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}\cdot p \ \Rightarrow\ \frac{3}{5} - \frac{1}{2} = \frac{2p-p}{2}$

$\Rightarrow\quad \frac{p}{2} = \frac{6-5}{10} \ \Rightarrow\ p = \frac{2}{10} = \frac{1}{5}$

**OR**

Let $A$ and $B$ be the events of getting a white pearl in the first draw and a black pearl in the second draw respectively.

Now, $P(A) = P$(getting a white pearl in the first draw)

$= \frac{12}{30} = \frac{2}{5}$

When second pearl is drawn without replacement, the probability that the second pearl is black is the conditional probability of the event $B$ occurring when $A$ has already occurred.

$\therefore\quad P(B\,|\,A) = \frac{18}{29}$

By multiplication rule of probability, we have

$P(A \cap B) = P(A) \cdot P(B\,|\,A) = \frac{2}{5} \times \frac{18}{29} = \frac{36}{145}$

**26.** We have, L.H.L. (at $x = 4$)

$= \lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{x-4}{|x-4|} + a = -1 + a$

R.H.L. (at $x = 4$)

$= \lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} \frac{x-4}{|x-4|} + b = 1 + b$

and $f(4) = a + b$

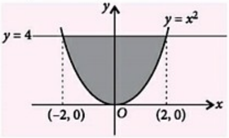
Since $f(x)$ is continuous at $x = 4$.

$\therefore \quad -1 + a = a + b \Rightarrow b = -1$

and $1 + b = a + b \Rightarrow a = 1$

Thus, $f(x)$ is continuous at $x = 4$ if $a = 1$ and $b = -1$.

**27.** We have, $y = x^2$ and $y = 4$



Required area = area of shaded region

$$= 2\int_0^2 (4 - x^2)dx = 2\left(4x - \frac{x^3}{3}\right)\Bigg|_0^2 = \frac{32}{3}\text{sq. units}$$

**28.** Given, $A = \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & 0 \\ -4 & 2 \end{vmatrix} = 2$ and adj $A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

Now, $(AB)^{-1} = B^{-1}A^{-1}$

$$= \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 10 & 1 \\ 18 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1/2 \\ 9 & 1 \end{bmatrix}$$

**OR**

We have,

$$\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} = \begin{vmatrix} \log_3 2^9 & \log_{2^2} 3 \\ \log_3 2^3 & \log_{2^2} 3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 9\log_3 2 & \frac{1}{2}\log_2 3 \\ 3\log_3 2 & \frac{2}{2}\log_2 3 \end{vmatrix} \quad \left[\because \log_{a^p} m^n = \frac{n}{p}\log_a m\right]$$

$$= (9\log_3 2) \times (\log_2 3) - \left(\frac{1}{2}\log_2 3\right)(3\log_3 2)$$

$$= 9(\log_3 2 \times \log_2 3) - \frac{3}{2}(\log_2 3 \times \log_3 2) = 9 - \frac{3}{2} = \frac{15}{2}$$

**29.** Given relation is $R = \{(a, b) : a \le b^2\}$

*Reflexivity:* Let $a \in$ real numbers.

$aRa \Rightarrow a \le a^2$

but if $a < 1$, then $a \nleq a^2$

For example, $a = \frac{1}{2} \Rightarrow a^2 = \frac{1}{4}$ so, $\frac{1}{2} \nleq \frac{1}{4}$

Hence, $R$ is not reflexive.

*Symmetricity* : $aRb \Rightarrow a \le b^2$

But then $b \le a^2$ is not true

$\therefore \quad aRb \not\Rightarrow bRa$

For example, $a = 2, b = 5$

then $2 \le 5^2$ but $5 \le 2^2$ is not true.

Hence, $R$ is not symmetric.

*Transitivity* : Let $a, b, c \in$ real numbers

Considering $aRb$ and $bRc$

$\quad aRb \Rightarrow a \le b^2$ and $bRc \Rightarrow b \le c^2$

$\Rightarrow \quad a \le c^4 \nRightarrow aRc$

For example, if $a = 2, b = -3, c = 1$

$\quad aRb \Rightarrow 2 \le 9$

$\quad bRc \Rightarrow -3 \le 1$

$\quad aRc \Rightarrow 2 \le 1$ is not true.

Hence, $R$ is not transitive.

**30.** Let $r$ be the radius of the base and $h$ be the height of a closed cylinder of given surface area $S$. Then,

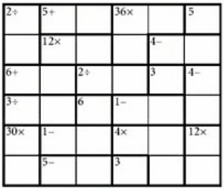
$$S = 2\pi r^2 + 2\pi rh \Rightarrow h = \frac{S - 2\pi r^2}{2\pi r} \qquad \text{...(i)}$$

## PUZZLE CORNER

### MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



| | | | | | |
|---|---|---|---|---|---|
| 2÷ | 5+ | | 36× | | 5 |
| | 12× | | | 4− | |
| 6+ | | 2÷ | | 3 | 4− |
| 3÷ | | 6 | 1− | | |
| 30× | 1− | | 4× | | 12× |
| | 5− | | 3 | | |

*Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.*

Let $V$ be the volume of the cylinder. Then,

$$V=\pi r^2h=\pi r^2\left(\frac{S-2\pi r^2}{2\pi r}\right)=\left(\frac{rS-2\pi r^3}{2}\right)$$

$$\Rightarrow \quad \frac{dV}{dr}=\frac{S}{2}-3\pi r^2$$

For maximum or minimum value of $V$, we have

$$\frac{dV}{dr}=0\Rightarrow \frac{S}{2}-3\pi r^2=0\Rightarrow S=6\pi r^2$$

From(i), $h=\dfrac{6\pi r^2-2\pi r^2}{2\pi r}\Rightarrow h=2r$

Also, $\dfrac{d^2V}{dr^2}=-6\pi r<0$

Hence, $V$ is maximum when $h = 2r$ *i.e.*, when the height of the cylinder is equal to the diameter of the base.

**31.** Let $u = e^x \sin x^3$ and $v = (\tan x)^x$

Now, $u = e^x \sin x^3$

Differentiating w.r.t. $x$, we get

$$\frac{du}{dx}=e^x\frac{d\left\{\sin(x)^3\right\}}{dx}+\sin x^3\frac{d}{dx}\,(e^x)$$

$$= e^x \cos x^3\, 3x^2 + \sin x^3\, e^x$$

Hence, $\dfrac{du}{dx} = 3x^2 e^x \cos x^3 + e^x \sin x^3$

Again, $v = (\tan x)^x \Rightarrow \log v = x \log (\tan x)$

Differentiating w.r.t. $x$, we get

$$\frac{1}{v}\frac{dv}{dx}=\log(\tan x)+x\frac{1}{\tan x}\sec^2 x$$

$$\therefore \quad \frac{dv}{dx} = v\,[\log (\tan x) + x \cot x \sec^2 x]$$

$$= (\tan x)^x\, [\log (\tan x) + x \cot x \sec^2 x]$$

Now, $y = u + v \;\Rightarrow\; \dfrac{dy}{dx}=\dfrac{du}{dx}+\dfrac{dv}{dx}$

$$\Rightarrow \quad \frac{dy}{dx} = 3x^2e^x\cos x^3 + e^x\sin x^3 + (\tan x)^x\,[\log(\tan x) + x \cot x \sec^2 x]$$

**OR**

We have, $x = 3 \sin t - \sin 3t$

$$\Rightarrow \quad \frac{dx}{dt} = 3 \cos t - 3\cos 3t \qquad \ldots\text{(i)}$$

$$y = 3 \cos t - \cos 3t \Rightarrow \frac{dy}{dt} = -3 \sin t + 3 \sin 3t \qquad \ldots\text{(ii)}$$

$$\therefore \quad \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\sin 3t-\sin t}{\cos t-\cos 3t} \qquad \text{[Dividing (ii) by (i)]}$$

$$=\frac{2\cos 2t\sin t}{2\sin 2t\sin t}=\cot 2t$$

Differentiating w.r.t. $x$, we get

$$\frac{d^2y}{dx^2}=-2\,\text{cosec}^2 2t\cdot\frac{dt}{dx}$$

$$= -2\,\text{cosec}^2 2t\cdot\frac{1}{3(\cos t-\cos 3t)} \qquad \text{[From (i)]}$$

At $t = \dfrac{\pi}{3}$,

$$\frac{d^2y}{dx^2}=-2\,\text{cosec}^2\frac{2\pi}{3}\cdot\frac{1}{3\left(\cos\frac{\pi}{3}-\cos\frac{3\pi}{3}\right)}$$

$$=-2\left(\frac{2}{\sqrt{3}}\right)^2\cdot\frac{1}{3\left(\frac{1}{2}+1\right)}=-\frac{16}{27}$$

**32.** Let $I=\displaystyle\int\frac{dx}{1-\cos x-\sin x}$

Since, $\cos x=\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\sin x=\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\therefore \quad I=\int\frac{dx}{1-\left(\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)-\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}=\int\frac{\frac{1}{2}\sec^2\frac{x}{2}dx}{\tan^2\frac{x}{2}-\tan\frac{x}{2}}$$

Put $\tan\dfrac{x}{2}=t \;\Rightarrow\; \dfrac{1}{2}\sec^2\dfrac{x}{2}dx=dt$

$$\therefore \quad I=\int\frac{dt}{t^2-t}=\int\left[\frac{1}{t-1}-\frac{1}{t}\right]dt$$

$$=\log(t-1)-\log t+C=\log\left|\frac{t-1}{t}\right|+C$$

$$=\log\left|\frac{\tan\frac{x}{2}-1}{\tan\frac{x}{2}}\right|+C=\log\left|1-\cot\frac{x}{2}\right|+C$$

**33.** Given, $f(0) = 2 \log 3 \qquad \ldots\text{(i)}$

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{(3^{\sin x}-1)^2}{x\log(1+x)}$$

$$=\lim_{x\to 0}\frac{\left(\dfrac{3^{\sin x}-1}{\sin x}\right)^2\cdot\left(\dfrac{\sin x}{x}\right)^2}{(1/x)\log(1+x)}$$

$$=\lim_{x\to 0}\frac{\left(\dfrac{3^{\sin x}-1}{\sin x}\right)^2\cdot\left(\dfrac{\sin x}{x}\right)^2}{\dfrac{\log(1+x)}{x}}$$

$$=\frac{(\log 3)^2 . (1)^2}{1}=(\log 3)^2 \qquad \ldots\text{(ii)}$$

From (i) and (ii), $\lim\limits_{x\to 0} f(x)\neq f(0)$

$\therefore$ $f$ is discontinuous at $x = 0$.

**OR**

We have, $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$

Here, $\frac{dx}{d\theta}=-2\sin\theta+2\sin 2\theta$

and $\frac{dy}{d\theta}=2\cos\theta-2\cos 2\theta$

$$\Rightarrow \frac{dy}{dx}=\frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}=\frac{2\cos\theta-2\cos 2\theta}{-2\sin\theta+2\sin 2\theta}$$

$$=\frac{\cos\theta-\cos 2\theta}{\sin 2\theta-\sin\theta}=\frac{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}=\tan\frac{3\theta}{2}$$

Differentiating w.r.t. $x$, we get

$$\frac{d^2y}{dx^2}=\sec^2\frac{3\theta}{2}\cdot\frac{3}{2}\frac{d\theta}{dx}=\frac{3}{2}\cdot\frac{1}{\cos^2\left(\frac{3\theta}{2}\right)}\cdot\frac{1}{-2\sin\theta+2\sin 2\theta}$$

$$=\frac{3}{4}\cdot\frac{1}{\cos^2\left(\frac{3\theta}{2}\right)}\cdot\frac{1}{2\cos\left(\frac{3\theta}{2}\right)\sin\frac{\theta}{2}}=\frac{3}{8}\frac{1}{\cos^3\left(\frac{3\theta}{2}\right)\sin\frac{\theta}{2}}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}}=\frac{3}{8\cos^3\left(\frac{3\pi}{4}\right)\sin\frac{\pi}{4}}=\frac{3}{8\left(-\frac{1}{\sqrt{2}}\right)^3\frac{1}{\sqrt{2}}}$$

$$=-\frac{3\times 4}{8}=-\frac{3}{2}$$

**34.** Given equation is $(x+y)^2\frac{dy}{dx}=a^2$

Put $x+y=u \Rightarrow 1+\frac{dy}{dx}=\frac{du}{dx}$

$$\therefore\ u^2\left(\frac{du}{dx}-1\right)=a^2 \Rightarrow u^2\frac{du}{dx}=u^2+a^2$$

$$\Rightarrow \int\frac{u^2}{u^2+a^2}du=\int dx \Rightarrow \int du-\int\frac{a^2du}{u^2+a^2}=x+C$$

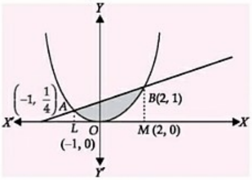
$$\Rightarrow u-a^2\times\frac{1}{a}\tan^{-1}\frac{u}{a}=x+C$$

$$\Rightarrow x+y-a\tan^{-1}\left(\frac{x+y}{a}\right)=x+C$$

$$\Rightarrow y-a\tan^{-1}\left(\frac{x+y}{a}\right)=C\Rightarrow y=a\tan^{-1}\left(\frac{x+y}{a}\right)+C$$

**35.** The given curve is $x^2 = 4y$ ...(i)

The given line is $x = 4y - 2$ ...(ii)



Putting $4y = (x + 2)$ from (ii) in (i), we get $(x + 2) = x^2$

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = 2, -1$

Putting $x = 2$ in (i), we get $y = 1$

Putting $x = -1$ in (i), we get $y = 1/4$

Thus the points of intersection of the given curve and line are $A\left(-1, \frac{1}{4}\right)$ and $B(2, 1)$.

$\therefore$ Required area

$$=\int_{-1}^{2}\left(\frac{x+2}{4}\right)dx-\int_{-1}^{2}\frac{x^2}{4}dx=\int_{-1}^{2}\left(\frac{x}{4}+\frac{1}{2}-\frac{x^2}{4}\right)dx$$

$$=\frac{1}{4}\left[\frac{x^2}{2}\right]_{-1}^{2}+\frac{1}{2}[x]_{-1}^{2}-\frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

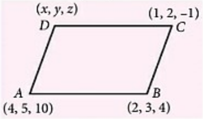
$$=\frac{1}{8}[4-1]+\frac{1}{2}[2+1]-\frac{1}{12}[8+1]$$

$$=\frac{3}{8}+\frac{3}{2}-\frac{3}{4}=\frac{3}{2}\left[\frac{1}{4}+1-\frac{1}{2}\right]=\frac{3}{2}\left[\frac{3}{4}\right]=\frac{9}{8}\text{sq. units}$$

**36.** In a parallelogram, diagonals bisect each other.

$\therefore$ Mid point of $BD$ = Mid point of $AC$

$$\Rightarrow \left(\frac{x+2}{2},\frac{y+3}{2},\frac{z+4}{2}\right)=\left(\frac{4+1}{2},\frac{5+2}{2},\frac{10-1}{2}\right)$$



$\Rightarrow x + 2 = 5, y + 3 = 7, z + 4 = 9 \Rightarrow x = 3, y = 4, z = 5$

So coordinates of $D$ are (3, 4, 5).

Cartesian equation of side $AB$ is

$$\frac{x-4}{2-4}=\frac{y-5}{3-5}=\frac{z-10}{4-10}$$

$$\Rightarrow \frac{x-4}{-2}=\frac{y-5}{-2}=\frac{z-10}{-6}\Rightarrow\frac{x-4}{1}=\frac{y-5}{1}=\frac{z-10}{3}$$

and vector equation of side $AB$ is

$\vec{r} = 4\hat{i}+5\hat{j}+10\hat{k} + \lambda(\hat{i}+\hat{j}+3\hat{k})$

Again, cartesian equation of side $BC$ is

$$\frac{x-2}{1-2} = \frac{y-3}{2-3} = \frac{z-4}{-1-4}$$

$$\Rightarrow \frac{x-2}{-1} = \frac{y-3}{-1} = \frac{z-4}{-5} \Rightarrow \frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{5}$$

and vector equation of side $BC$ is

$\vec{r} = 2\hat{i}+3\hat{j}+4\hat{k} + \lambda(\hat{i}+\hat{j}+5\hat{k})$

**OR**

*Refer to Answer 112, page no. 278-279 of MTG CBSE Champion Mathematics, Class 12.*

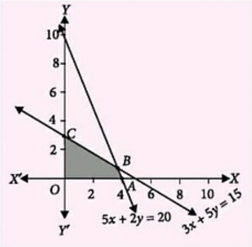
**37.** Converting inequations into equations and drawing the corresponding lines.

$3x + 5y = 15,\ 5x + 2y = 20$

*i.e.* $\frac{x}{5}+\frac{y}{3}=1,\ \frac{x}{4}+\frac{y}{10}=1$

As $x \geq 0, y \geq 0$ solution lies in first quadrant.

Let us draw the graph of the above equations.



$B$ is the point of intersection of the lines $3x + 5y = 15$ and $5x + 2y = 20$, *i.e.* $B = \left(\frac{70}{19}, \frac{15}{19}\right)$

We have points $O(0, 0)$ $A(4, 0)$, $B\left(\frac{70}{19}, \frac{15}{19}\right)$ and $C(0, 3)$

Now $z = 3x + 2y$

$\therefore\ z(O) = 3(0) + 2(0) = 0$

$z(A) = 3(4) + 2(0) = 12$

$z(B) = 3\left(\frac{70}{19}\right)+2\left(\frac{15}{19}\right) = 12.63$

$z(C) = 3(0) + 2(3) = 6$

$\therefore$ $z$ has maximum value 12.63 at only one point *i.e.* $B\left(\frac{70}{19}, \frac{15}{19}\right)$

**OR**

*Refer to Answer 6, page no. 291 of MTG CBSE Champion Mathematics, Class 12.*

**38.** We have, $A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$

$\Rightarrow |A| = 3(-16 + 8) + 10(4 - 4) - 1(8 - 16) = -24 + 8$
$= -16 \neq 0$. So, $A^{-1}$ exists

$$\therefore\ \text{adj } A = \begin{bmatrix} -8 & -16 & -12 \\ 0 & -4 & -4 \\ -8 & -8 & 4 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{|A|}.(\text{adj } A)$

$$= \frac{-1}{16}\cdot\begin{bmatrix} -8 & -16 & -12 \\ 0 & -4 & -4 \\ -8 & -8 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 & \frac{3}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$$

$$\text{Now, } (A^{-1})^2 = \begin{bmatrix} \frac{1}{2} & 1 & \frac{3}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{4} \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 1 & \frac{3}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{8} & \frac{9}{8} & \frac{7}{16} \\ \frac{1}{8} & \frac{3}{16} & 0 \\ \frac{1}{8} & \frac{1}{2} & \frac{9}{16} \end{bmatrix}$$

**OR**

*Refer to Answer 98, page no. 70-71 of MTG CBSE Champion Mathematics, Class 12.*

◆◆

## Monthly Test Drive-10 CLASS XI ANSWER KEY

**1.** (d) **2.** (c) **3.** (a) **4.** (a) **5.** (c)
**6.** (c) **7.** (c) **8.** (a, b) **9.** (a, b) **10.** (c)
**11.** (a, c) **12.** (a, c) **13.** (a, d) **14.** (d) **15.** (b)
**16.** (b) **17.** (9) **18.** (5) **19.** (2) **20.** (5)

# MONTHLY TEST DRIVE

## XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

**Total Marks : 80**

## Series 10 : Statistics and Probability

**Time Taken : 60 Min.**

### Only One Option Correct Type

**1.** If $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of the three mutually exclusive events, then $p \in$

(a) $[0, 1]$ (b) $\left[0, \frac{1}{2}\right]$ (c) $\left[\frac{1}{3}, 1\right]$ (d) $\left[\frac{1}{3}, \frac{1}{2}\right]$

**2.** Mean of 100 items is 49. It was discovered that 3 items 60, 70, 80 were wrongly read as 38, 22, 50 respectively. The correct mean is

(a) 48 (b) 78 (c) 50 (d) 80

**3.** The standard deviation of 25 numbers is 40. If each of the number is increased by 5, then the new standard deviation will be

(a) 40 (b) 25 (c) $\sqrt{40}$ (d) 1600

**4.** If $P(B) = \frac{3}{4}, P(A \cap B \cap \bar{C}) = \frac{1}{3}$ and $P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$, then $P(B \cap C) =$

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{15}$ (d) $\frac{1}{9}$

**5.** Two dice are thrown simultaneously to get the coordinates of a point on $xy$-plane. Then the probability that this point lies inside or on the region bounded by $|x| + |y| = 3$, is

(a) $\frac{3}{14}$ (b) $\frac{2}{3}$ (c) $\frac{1}{12}$ (d) $\frac{4}{14}$

**6.** Out of $(2n + 1)$ tokens consecutively numbered from 1 to $(2n + 1)$, 3 tokens are drawn at random. The probability that the numbers on them are in A.P. is

(a) $\frac{2n}{4n^2-1}$ (b) $\frac{4n}{4n^2-1}$ (c) $\frac{3n}{4n^2-1}$ (d) $\frac{5n}{4n^2-1}$

### One or More Than One Option(s) Correct Type

**7.** If the mean deviation of numbers $1, 1 + d, 1 + 2d, \ldots, 1 + 100d$ from their mean is 255, then the $d$ is equal to

(a) 10 (b) 20 (c) 10.1 (d) 20.2

**8.** The variable $x$ takes two values $x_1$ and $x_2$ with frequencies $f_1$ and $f_2$, respectively. If $\sigma$ denotes the standard deviation of $x$, then

(a) $\sigma^2 = \frac{f_1x_1^2 + f_2x_2^2}{f_1 + f_2} - \left(\frac{f_1x_1 + f_2x_2}{f_1 + f_2}\right)^2$

(b) $\sigma^2 = \frac{f_1f_2}{(f_1 + f_2)^2}(x_1 - x_2)^2$

(c) $\sigma^2 = \frac{(x_1 - x_2)^2}{(f_1 + f_2)^2}$ (d) none of these

**9.** Which of the following is correct statement?

(a) The sum of the deviations from arithmetic mean is zero.

(b) Computation of arithmetic mean is based on all observations.

(c) Arithmetic mean gives no importance to extreme values.

(d) none of these

**10.** The probability that the 13th day of a randomly chosen month is a second Saturday, is

(a) $\frac{1}{7}$ (b) $\frac{1}{12}$ (c) $\frac{1}{84}$ (d) $\frac{19}{84}$

**11.** The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are

(a) $p_1 = 1/9$ (b) $p_1 = 1/16$

(c) $p_2 = 1/3$ (d) $p_2 = 1/4$

**12.** The variance of the numbers 2, 3, 11 and $x$ is $\frac{49}{4}$. Find the value of $x$.

(a) 6 (b) $\frac{14}{5}$ (c) $\frac{14}{3}$ (d) $\frac{6}{15}$

**13.** If runs of two players $A$ and $B$ in 10 cricket matches are such that player $A$ has mean 50 and variance 36 and player $B$ has mean 60 and variance 81 of runs, then the player

(a) $A$ has less coeff. of variation
(b) $B$ has less coeff. of variation
(c) Both are equally consistent
(d) $A$ is more consistent

### Comprehension Type

In an objective paper, there are two sections of 10 questions each. For 'section 1', each question has five options and only one option is correct and 'section 2' has four options with multiple answers and marks for a question in this section is awarded only if he ticks all correct answers. Marks for each question in 'section 1' is 1 and in 'section 2' is 3. (There is no negative marking).

**14.** If a candidate in total attempts four questions all by guessing, then the probability of scoring 10 marks is

(a) $1/15(1/15)^3$ (b) $3/5(1/15)^3$
(c) $1/15(14/15)^3$ (d) None of these

**15.** The probability of getting a score less than 40 by answering all the questions by guessing in this paper is

(a) $(1/75)^{10}$ (b) $1-(1/75)^{10}$
(c) $(74/75)^{10}$ (d) None of these

### Matrix Match Type

**16.** Match the following.

| | Column I | | Column II |
|---|---|---|---|
| P. | There are 5 duplicate and 10 original items in an automobile shop and 3 items are brought at random by a customer. The probability that none of the items is duplicate, is | 1. | $\frac{1}{6}$ |
| Q. | There are 2 teams with $n$ persons in each. The probability of selecting 2 persons from one team and 1 person from the other team is $\frac{6}{7}$. Then $\frac{1}{n}=$ | 2. | $\frac{24}{91}$ |
| R. | If the letters of the word NALGONDA are arranged in arbitrary order, the probability that the letters G, O, D appear in that order is | 3. | $\frac{10}{19}$ |
| S. | Out of 20 positive consecutive integers, two are chosen at random. The probability that their sum is odd, is | 4. | $\frac{1}{4}$ |

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 2 | 1 | 3 | 4 |
| (b) | 2 | 4 | 1 | 3 |
| (c) | 3 | 1 | 4 | 2 |
| (d) | 3 | 4 | 1 | 2 |

### Numerical Answer Type

**17.** A card is drawn from a pack of 52 cards and a gambler bets that it is a spade or an ace. If the odds against his winning this bet is $\frac{k}{k-5}$, then the value of $k$ is ______.

**18.** The A.M. and S.D. of 100 items was recorded as 40 and 5.1 respectively. Later on it was discovered that one observation 40 was wrongly copied down as 50. Then the correct S.D is ______.

**19.** The mean and variance of 7 observations are 8 and 16 respectively. If the observations are 2, $x$, 4, 10, 12, $y$, 14. Then the positive value of $x-y$ is ______.

**20.** Let $A=\{1, 3, 5, 7, 9\}$ and $B=\{2, 4, 6, 8\}$. An element $(a,b)$ of their cartesian product $A \times B$ is chosen at random. If the probability that $\{a+b=9\}$ is $\frac{p}{q}$, where $p$ and $q$ are coprime numbers, then $\frac{q}{p}$ is ______.

❖❖

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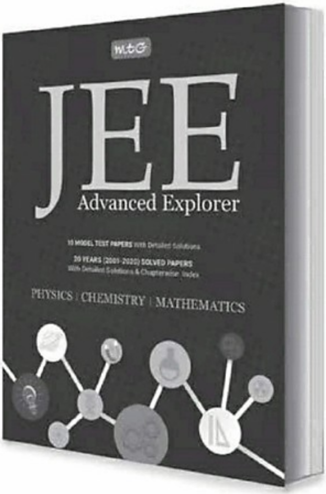
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# MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

**Total Marks : 80**

## Series 10 : Probability

**Time Taken : 60 Min.**

### Only One Option Correct Type

**1.** A determinant is chosen at random from the set of all determinants of order 2 with elements 1 or 0 only. The probability that the value of the determinant is positive is

(a) $\frac{1}{4}$ (b) $\frac{2}{9}$ (c) $\frac{3}{16}$ (d) $\frac{1}{10}$

**2.** A bag *A* contains 2 white and 3 red balls and bag *B* contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. The probability that it was drawn from bag *B* is

(a) 25/52 (b) 13/27 (c) 8/17 (d) 25/51

**3.** If $X$ and $Y$ are independent binomial variates $X\left(5, \frac{1}{2}\right)$ and $Y\left(7, \frac{1}{2}\right)$, then $P(X + Y = 3)$ is

(a) $\frac{55}{1024}$ (b) $\frac{55}{4098}$

(c) $\frac{55}{2048}$ (d) None of these

**4.** If the integers $\lambda$ and $\mu$ are chosen at random between 1 to 100, then the probability that a number of the form $7^{\lambda} + 7^{\mu}$ is divisible by 5, is

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{49}$

**5.** A natural number $x$ is chosen at random from the first hundred natural numbers. The probability that $\frac{(x-20)(x-40)}{(x-30)} < 0$ is

(a) $\frac{1}{50}$ (b) $\frac{3}{50}$ (c) $\frac{7}{25}$ (d) $\frac{9}{50}$

**6.** Three natural numbers are taken at random from the set $A = \{x \mid 1 \leq x \leq 100, x \in N\}$. The probability that the A.M. of the numbers taken is 25, is

(a) $\frac{{}^{77}C_2}{{}^{100}C_3}$ (b) $\frac{{}^{25}C_2}{{}^{100}C_3}$ (c) $\frac{{}^{74}C_{72}}{{}^{100}C_{97}}$ (d) $\frac{{}^{75}C_2}{{}^{100}C_3}$

### One or More Than One Option(s) Correct Type

**7.** Three numbers are chosen at random without replacement from $\{1, 2, ..., 15\}$. Let $E_1$ be the event that minimum of the chosen numbers is 5 and $E_2$ their maximum is 10 then

(a) $P(E_1) = \frac{9}{91}$ (b) $P(E_2) = \frac{36}{455}$

(c) $P(E_1 \cap E_2) = \frac{4}{455}$ (d) $P(E_1 / E_2) = \frac{1}{9}$

**8.** A fair coin is tossed 99 times. If $r$ is the number of times tail occurs, then $P(X = r)$ is maximum when $r$ is equal to

(a) 49 (b) 51

(c) 50 (d) None of these

**9.** Let $X$ be a set containing $n$ elements. If two subsets $A$ and $B$ of $X$ are picked at random, the probability that $A$ and $B$ have the same number of elements is

(a) $\frac{{}^{2n}C_n}{2^{2n}}$ (b) $\frac{1}{{}^{2n}C_n}$

(c) $\frac{1 \cdot 3 \cdot 5 \cdot ... \cdot (2n-1)}{2^n \cdot n!}$ (d) $\frac{3^n}{4^n}$

**10.** Let $X$ and $Y$ be two events such that

$$P(X \mid Y) = \frac{1}{2}, P(Y \mid X) = \frac{1}{3} \text{ and } P(X \cap Y) = \frac{1}{6}.$$

Which of the following is(are) correct?

(a) $P(X \cup Y) = \frac{2}{3}$

(b) $X$ and $Y$ are independent

(c) $X$ and $Y$ are not independent

(d) $P(X^c \cap Y) = \frac{1}{3}$

**11.** A coin is tossed $(2n + 1)$ times, the probability that head appear odd number of times is

(a) $\frac{n}{2n+1}$ (b) $\frac{n+1}{2n+1}$ (c) $\frac{1}{2}$ (d) $\frac{n!}{2n+1}$

**12.** A card is selected at random from cards numbered as 00, 01, 02, ...., 99. An event is said to have occured, if product of digits of the card number is 16. If cards is selected 5 times with replacement each time, then the probability that the event occurs exactly three times is

(a) ${}^5C_3\left(\frac{3}{100}\right)^2\left(\frac{97}{100}\right)^3$ (b) ${}^5C_3\left(\frac{3}{100}\right)^3\left(\frac{97}{100}\right)^2$

(c) ${}^5C_3\left(\frac{0.3}{100}\right)^3\left(\frac{9.7}{100}\right)^3$ (d) $10(0.03)^3(0.97)^2$

**13.** A coin is tossed repeatedly. $A$ and $B$ call alternately for winning a prize of ₹ 30. One who calls correctly first wins the prize. $A$ starts the call. Then, the expectation of

(a) $A$ is ₹ 10 (b) $B$ is ₹ 10

(c) $A$ is ₹ 20 (d) $B$ is ₹ 20

## Comprehension Type

The probability of happening of an event in one trial being known, then the probability of its happening exactly $x$ times in $n$ trials is given by ${}^nC_x\, q^{n-x} \cdot p^x$ where
$p$ = probability of happening the event
$q$ = probability of not happening the event = $1 - p$
Also, ${}^nC_x\, q^{n-x}\, p^x$ is $(x + 1)^{\text{th}}$ term in the expansion of $(q + p)^n$ whose expansion gives the happening of the event 0, 1, 2, ...., $n$ times, respectively.

**14.** In four throws with a pair of dice, the chance of throwing doublets atleast twice is

(a) $\frac{19}{144}$ (b) $\frac{125}{144}$ (c) $\frac{17}{144}$ (d) $\frac{18}{144}$

**15.** Unbiased coin is tossed 6 times. The probability of getting atmost 4 heads is

(a) $\frac{7}{64}$ (b) $\frac{57}{64}$ (c) $\frac{21}{32}$ (d) $\frac{11}{32}$

## Matrix Match Type

**16.** Sixteen players $S_1, S_2, ...., S_{16}$ play in a tournament. They are divided into eight pairs at random. From each pairs, a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength.

| | Column I | | Column II |
|---|---|---|---|
| P. | The probability that player $S_1$ is among the eight winners is | 1. | 4/5 |
| Q. | If the probability that exactly one of $S_1$ and $S_2$ is among the eight winners is $\frac{2\lambda}{3}$, where $\lambda$ equal to | 2. | 7/30 |
| R. | The probability that both $S_1$ and $S_2$ are among the eight winners is | 3. | 1/2 |

| | P | Q | R |
|---|---|---|---|
| (a) | 3 | 1 | 2 |
| (b) | 3 | 2 | 1 |
| (c) | 1 | 2 | 3 |
| (d) | 2 | 1 | 3 |

## Numerical Answer Type

**17.** Let $A$ and $B$ be two events such that $P(A') = 0.3$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$, then $P(B|A \cup B') = \frac{2}{n}$, where $n$ is ______.

**18.** If four squares are chosen at random on a chess board. If the probability that they lie on a diagonal line is $\frac{\lambda}{{}^{64}C_4}$, then the unit place of $\lambda$ is ______.

**19.** The probability that the roots of the equation $x^2 + nx + n/2 = 0$ are real, $n \in N$ and $n \le 5$, is $m/n$,then $m$ = ______.

**20.** $A$, $B$, $C$ and $D$ cut a pack of 52 cards successively in the order given. Also, the person who cuts a spade first receives ₹ 350. If $E(A)$ denotes the expected amount of $A$, then the value of $\frac{E(A)}{64}$ is ______.

◆◆

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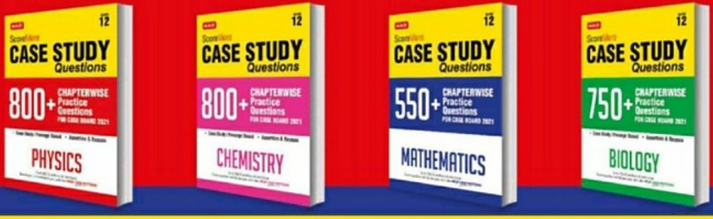
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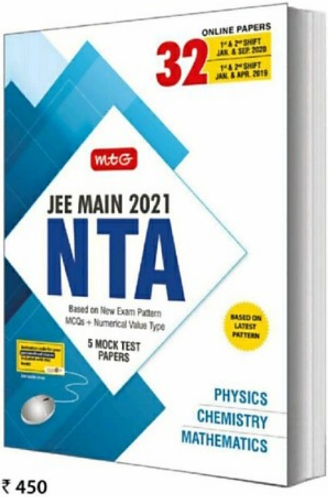
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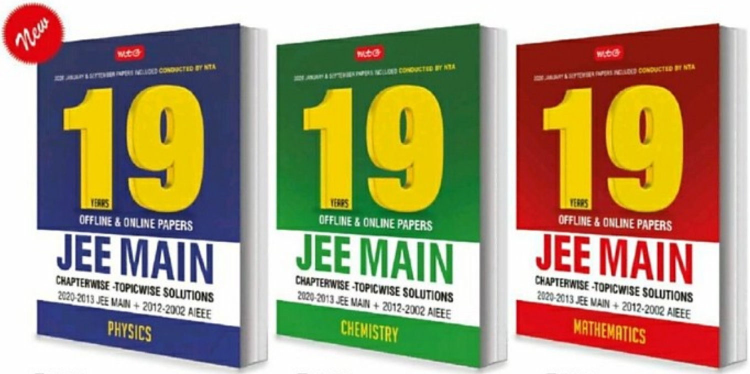
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